Inquiry in University Mathematics Teaching and Learning: The PLATINUM Project
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CHAPTER 19

Epilogue

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The production of this book is the last stage in a journey which 31 authors, belonging to the 8 university institutions, partners of the PLATINUM Project, have shared focusing on Inquiry-Based Mathematics Education (IBME). The journey has been a recurring metaphor in an attempt to illustrate the meaning of life. We find the term in the displacement from one territory to another, the walk that separates a departure from an arrival, as a space to give meaning to what has been lived. The metaphor of the journey thus becomes a commonplace worth analysing and so we wish to use it to synthesise the lessons learned from the production of this book. In the following paragraphs we will take a brief tour, like someone moving her finger to find herself on a map, to analyse this commonplace that we recognise as valuable for the university community in mathematics.

19.1. Conceptualisation of Inquiry at University Level

It would be pertinent to look back on the content page in Inquiry in University Mathematics Teaching and Learning. In Part 1 about conceptual foundations of the PLATINUM Project, taking an inquiry perspective on mathematics education we have tried to specify what exactly we mean by inquiry. In our definition of inquiry, we used the idea of emulating mathematicians: seeking to know, exploring, investigating, finding out, asking questions, solving problems, looking critically, developing Inquiry "as a way of being." In Part 2, we explained and provided examples of how we addressed the processes and principles with which we engaged in our PLATINUM activity. With all chapters in Part 3, all cases of the partners' development of inquiry-based teaching, we set out to explain our approach and how to help others achieve it. What we have done is to set out on a journey in which the reader has been encouraged to participate, to consider the way in which Inquiry processes can take place in University Mathematics Teaching and Learning. The reader can find examples in the fields of analysis, algebra, statistics, etc. to understand what makes some experiences successful and help others to act as mathematical thinkers on their learning journey.

For us, as editors, it has been an interesting study to see the development of various dimensions of an approach to inquiry in mathematics. One that has reinforced engagement and enhanced learning is the concept of inquiry community. Inquiry communities permeated PLATINUM activity at all levels from small groups of colleagues working together to develop teaching, to the whole PLATINUM community working together to produce this book. We recognise this throughout the chapters. Although we all agreed on the support of these dimensions, the underlying theoretical frameworks in mathematics education offered different perspectives and nuances. This is unsurprising since we come from different cultures and different traditions. Although

it has led to differences along the way, requiring a rapprochement of positions and a deeper grounding of our original proposal, we have recognised it as valuable. It has brought richness and depth to the project and enhanced our personal conceptions of learning and teaching. (Chapters 3, 7 and 9 provide examples of this).

19.2. Pathways in the Design of Materials in University Mathematics

Chapter 4 of Part 1 and the chapters in Part 2 provide insights into the learning avenues experienced in the design of materials for teaching and learning under the inquiry approach. From our perspective, what counts are the actions developed in teaching: This means teaching in ways that result in students' developing deep understandings of mathematics as a subject matter.

First: We have, individually and collectively, gained ideas about productive teaching practices in inquiry-based learning.

Second: Since a great deal of current classroom practice does not match our ideas about productive teaching practices at university level, we would like to see changes, first of all in our own professional practice. This book represents progress in our understandings of the complex construct of inquiry approach at university level which we hope readers will find valuable for their own teaching.

Third: The commitment to social integration in the educational institutions of the partners has encouraged breaking down barriers and ensuring an inclusive learning environment so that students with identified needs are able to participate "independently and equally." Although all the partners have experience of teaching students with identified (special) needs, the specialised focus, presented in Chapter 4 considering the diversity of students, has influenced units and tasks in the courses that were designed.

In the creating teaching units for student inquiry, different authors pointed out several emphases that have become in the context of IBME:

(1) the 'authenticity' of inquiry questions, the connection of students' activities with their real life;
(2) the epistemological relevance of inquiry questions from a mathematical perspective in the statements and in the guides formulated by the teacher;
(3) the experimental and applied dimensions of mathematics and the interdisciplinary knowledge that this demands;
(4) the collaborative dimension of the inquiry process;
(5) student diversity as a cross-cutting issue to be taken into account alongside developing understanding of addressing identified needs.

We learn from the exploration of these ways of teaching that promote students' inquiry-based learning of mathematics. We look at where we were now in terms of lectures, didactic and pedagogic processes. Based on these experiences, we are more aware of how we can use inquiry-based processes to help students engage with mathematics more conceptually. We become familiar with exploring and developing our own practices, which is itself an inquiry process. So, in the formal presentation of material to students in university mathematics in this volume—including mathematics majors and mathematics as a service subject—we recognise conceptual obstacles that make the pathway very difficult for students to travel successfully. There are still avenues that will require further deepening in the future: inquiry with technology, inquiry and modelling, inquiry and algorithmic processes, inquiry and interdisciplinary projects.

19.3. Methods and Materials for Professional Development of Lecturers

As stated in Chapters 7 and 16, the PLATINUM team's idea of 'professional development' is based on an epistemology of professional knowledge which takes into account the contextualised nature of the lecturer's experience (experience knowledge) and the personalised knowledge of the practice.

In the development of this project and in the preparation of this book there has been a continuous dialogue between local and universal perspectives. The countries that take part have different cultural and social contexts even though we are all in the university environment, the different mathematics and mathematics education traditions. Community of Inquiry and the associated concept of Critical Alignment have been central to both the theoretical and practical aspects of PLATINUM. Community of Inquiry (CoI) can be seen to derive from CoP (community of practice) where the 'alignment' requirement of CoP is developed to become 'Critical Alignment.' This is mentioned (briefly) in Chapter 2. In the PLATINUM project, in relation to the professional development of lecturers, we took an expansive view of the notion, where there are no geographical boundaries to such communities.

Members of these communities have developed their teaching in certain ways, both personal and institutional. In the experience of local and international workshops for professional development lecturers highlighted that teaching decisions and actions are not 'just' actions, they manifest a wealth of knowledge, goals, and orientations. Lecturers, being members of the large community of educative practice, have been enculturated into a set of pedagogical and didactical assumptions (that is, beliefs and orientations) that shape their practice in mathematics education and which have varied in accord with the different national cultures.

A challenge in PLATINUM was the creation of nested communities of inquiry even if they came from different countries with respect to several issues essential to the project's aims at university level (conceptualisation of inquiry, teacher professional development and assessment). The achievement of joint work in these communities can be in the production of the different intellectual outputs whose work is reflected in different chapters. For example, in Part 2 of the book: in Chapter 7 the joint work on teacher education of Norway, Germany, Netherlands and Spain, in Chapter 8 the joint work on mathematical modelling and inquiry-based learning of Norway, Czech Republic and Ukraine, or in Chapter 9 the joint work of Czech Republic, Netherlands, Spain and UK on methods and instruments in assessment from the perspective of inquiry-based mathematics education. This is important if we want to think about lecturer/teacher change, in that it helps to identify the orientations of lecturers which are valuable in this perspective or which should be changed.

We would like to point out that, in PLATINUM, inquiry-based practice is itself an important source of professional development as can be seen in the chapters in Part 3. We worked together during more than three years, we are all more knowledgeable in what inquiry means for us, in what we can do to engage with inquiry, and the differing ways in which we can engage. If readers of this book or new colleagues join us, we can draw them into our communities and they can learn through working alongside others with critical alignment.

19.4. And to Conclude

Our project aims to be ambitious in promoting a classroom culture change and lecturer identity from the IBME approach to teaching and learning. Much of the literature in IBL, IBME etc. focuses on inquiry in mathematics. In PLATINUM,

this is largely the central layer of the model (Chapters 2 and 3). We extend it into the middle layer as lecturers think about and design mathematical tasks for students' inquiry-based activity. The PLATINUM model extends much further to the process of inquiring into learning and teaching and the associated development that comes with this. We learn from the experience, to ask questions, to share with colleagues, to develop our own practice through this process, and to research the outcomes. In our journey, in order to systematise the experience and to become more systematic and rigorous, through critical alignment, we have been engaged in what we call 'Developmental Research.' Developmental research is research that both studies the developmental process and also contributes to the development itself. It develops new knowledge in both theory and practice, encapsulated within the three-layer model. This is the uniqueness of the PLATINUM model.

We point out this dimension of novelty and uniqueness of the PLATINUM model within scientific production because a successful change in university teaching requires both a set of new teaching techniques and a constellation of orientations (about mathematics, about concepts, about what students can do, and about classroom practices) in order to take hold. Change in improving teaching can be long and slow journey (it has taken us more than three years), but it is worth it. We hope that the contributions in this volume have offered progress towards this goal.