Inquiry in University Mathematics Teaching and Learning: The PLATINUM Project
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CHAPTER 14

# In Critical Alignment With IBME

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## 14.1. Introduction

The aim of this contribution is to describe the professional growth of the Leibniz-University-Hannover-group (LUH-group). The four authors of this chapter are the core members of the LUH-group[1] and belong to a working group in the mathematics education department of Leibniz University Hannover. In the beginning, a number of people from other departments were interested in joining the LUH-group, but either transferred to other universities (in Germany staff changes between universities are quite common), or didn't find the time to attend meetings on a regular basis. The LUH-group conducts research in the field of university mathematics education and, with regard to teaching, offers courses in mathematics education for prospective mathematics teachers at secondary school level. This means all LUH-group-members are mathematics education researchers as well as mathematics teacher educators. The reported professional growth is connected to our involvement in and reflection of a developmental research project called Leibniz-Prinzip (see Section 14.2).

We take this project and observations that we made in connection with it as starting point and develop from this a reflection on central theoretical foundations of PLATINUM regarding its concept of *Inquiry-based Mathematics Education* (IBME). Potentials and goals of cooperative development addressed in the concept of *Community of Inquiry* (CoI) will be questioned with regard to their implicit assumptions, prerequisites and conditions for success. Drawing on our local conditions and experiences, we will critically examine these implicit assumptions, prerequisites and conditions for success, which can be understood as forms of personal and institutional specifications of our current prerequisites and potentials for further development. For us, the idea of CoI functions as a counter-horizon[2] against which restrictive conditions and potentials for further development will be fleshed out in more detail (see Section 14.4). The initial idea of CoI articulated in the three-layer-model (see Chapter 2) hints at an interrelatedness and mutual enrichment of developmental research and professional growth. The three-layer-model indicates that the members of the PLATINUM project, in their activities of fostering IBME, are simultaneously involved in different but interrelated

[1]Because we consider *Community of Inquiry* to be an analytical concept, we refer to ourselves (as people) with the term *(LUH-)group*. The LUH-group, of course, can be analysed concerning its characteristics regarding the analytical concept. Any critique in this contribution refers exclusively to the analytical concept of "Community of Inquiry," and not to the work of PLATINUM groups published in this book.

[2]The term 'counter-horizon' refers to a horizon of interpretation that opposes typical horizons referred to in a field of practice. We detail which typical horizons we oppose in Section 14.4.1. The counter-horizon is regarded as an alternative outlook among a number of more common or widespread interpretations.

CoIs. These CoIs can be described with reference to the positions of their members, their inquiry interests and the objective of their inquiry activities, see Table 14.1.

| **Layer** | **Positions** | **Inquiry into...** | **objective** |
|---|---|---|---|
| inner layer | students and teacher(s) | mathematical activity | learning mathematics |
| middle layer | teachers | teaching and learning practices | professional growth |
| outer layer | teachers and researcher(s) | IBME | developmental research |

TABLE 14.1. Positions of members, the scope of inquiry and the objective of interrelated CoI.

Concerning the positions addressed in the three-layer-model, all group members within our LUH-group simultaneously occupy the positions of teachers and researchers. Having this double-responsibility can be regarded as typical for German universities and university teacher education specifically (e.g. Adler et al., 2005).[3] In this situation, university teachers[4] are often said to be "in a double role." We would argue, though, that the understanding of the positions in CoIs needs to go beyond interpreting them as roles: Common role concepts harbour the danger of (1) subordinating positions and their scope for action to institutional goals and conditions, (2) personalising contradictions and barriers and (3) demanding a professionalisation of persons that aims at a mere satisfaction of role requirements. All three aspects entail an ignorance of contradictions in the institutional-social contexts of reference. Such a mode of thinking, which personalises contradictions, can furthermore be a symptom of an inadequate theoretical analysis of positions (for a reflection of our positions as teachers and researchers see Ruge & Peters, 2021). Thus, challenging common role concepts in teaching-learning relations and professional development might be a starting point for questioning and extending concepts of learning and development. Such possible extensions can be found, for example, in Engeström's (1987) concept of expansive development and in Holzkamp's (1995, 2013) understanding of agency. The concept of expansive development primarily addresses institutional-systemic dimensions of development.[5] Holzkamp's understanding of agency opens up ways to explore the personal possibilities to act within or upon conditions. The relation between personal possibilities and underlying conditions is conceptualised with reference to their societal-mediatedness and historical specificity. This understanding of agency is an

[3]However, staff members in Germany are usually employed on fixed-term contracts that privilege research activities over teaching, both in terms of allocated time and of criteria for promotion and further employment. This creates tensions and contradictions when trying to fulfil both responsibilities.

[4]Unless otherwise specified, we use the term 'teacher(s)' to denote university teachers, 'student(s)' refers to university students (in our case teacher students), and 'pupil(s)' is used for secondary school students.

[5]Engeström proposed an analytical tool for the description of activity systems and further developments of the object of an activity. The activity system is described from the point of view of an individual or a subgroup and integrates the community, its division of labour and rules in the analysis of the development of the object. The development of activity systems is judged by the development of the object, the formation of a new or expanded object (Sannino & Engeström, 2018). Therefore, the focus is on the systemic level and suitable to describe developmental processes of organisations or institutions.

important point of reference of our research activities in the LUH-group (for details see Section 14.4.2) and serves us as basis for reflections and debates.

A central topic of the reflections and debates within our group was the course "Introduction to Mathematics Education," in which three of the team members were involved as teachers. The participants of this course are prospective mathematics teachers at secondary school level who are still at the beginning of their university studies. The focus of the course is on fostering reflective agency (see Section 14.2), that is based on Holzkamp's understanding of agency. In the development and implementation of the course, phenomena arose that we reflected and debated on within our group. The phenomena can be understood as manifestations of conflicts and areas of tension that are typical within our context. In particular we address phenomena that are described in the literature under the headings of *theory vs. practice* (Terhart, 2000), *teaching-learning short-circuit vs. guidance* (Holzkamp, 1995; Huck, 2013), and *autonomy-antinomy* (Helsper, 1996). Our reflections on and debates of these phenomena led us to rethink the concepts of CoI and IBME and their initial framing in PLATINUM. We enrich, differentiate and modify them, to rearticulate the potentials that we claim these concepts hold for the further development of theory and teaching practice (see Section 14.4). To us, these aspects are strongly connected to each other. This is reflected in our mode of participation in PLATINUM, which can be described as a constant back and forth between further development of theory and further development of teaching practice. Both contribute to each other. We want to detail this reciprocity of development in theory and teaching practice, which characterises the process of our joint professional growth and forms the core of our development.

We structured our contribution as follows: In Section 14.2, we first describe the context of the teaching project of our group – the developmental research project Leibniz-Prinzip and the course "Introduction to Mathematics Education," which was developed in this project. We briefly describe the course and its overall goal of fostering reflective agency, before we provide a sample task with a description of the particular content to be inquired into and outline experiences with student reactions and solutions. In Section 14.3, we describe the above-mentioned selected phenomena and contradictions (theory vs. practice, teaching-learning short-circuit vs. guidance, autonomy-antinomy) in the context of this teaching project. We describe our engagement with these phenomena with reference to the theoretical foundations of the concept of reflective agency and then reflect on consequences for our development as LUH-group, i.e., the process of our professional growth. In formulating these descriptions and our interpretations, we consider ourselves in the position of teachers and researchers in the local context of the LUH-group. In Section 14.4, we contemplate the process of our professional growth against the background of the concepts of IBME and CoI in PLATINUM. On a meta-level we reflect on our experiences as teachers and researchers within our local PLATINUM project. Within the global PLATINUM group we also consider ourselves researchers and take up this position for our formulations of a further development of the theoretical foundations of the PLATINUM project. We will present reflections which, among other things, point to the necessity of both the conceptual concretisation of the three-layer-model and the consideration of societal and professional aspects. Our conclusion suggests a restriction of the goals of IBME and an expansion of the concept of critical alignment, which is described in (Jaworski, 2006) as

> ... critiquing and trying to develop, improve or enhance the status quo, alongside enculturation into existing social norms. However, the significance of normal desirable states is just that they are desirable within the social practices in which they have developed.

> It is hard to operate against such practices, or to challenge them in practice. [...] I see the term "critical," in "critical alignment," as indicating a key concept for avoiding the perpetuation of undesirable states. (p. 191)

Within this contribution, we seek to detail our critical alignment. From our point of view, Section 14.4 covers the core of our case description.

## 14.2. Context of the Teaching Project of the LUH-group: The Course, the Concern for Reflective Agency, and the Sample Task

Our professional practice as mathematics teacher educators is situated at the beginning of the first phase of teacher education, which in Germany has a three-phase structure: The first phase is the university study programme, the second phase is preparatory service, and the third phase is in-service training. The university phase is commonly considered to be "more theoretical" while preparatory service in schools and seminars and further in-service training are thought to be "more practical." Traditionally, the German educational system insists upon an academic education of teachers, particularly for teachers of secondary schools.[6] Prospective secondary school teachers typically study two subjects at university and complete mostly the same courses as regular Bachelor students of the respective subject. Additionally, they have to take courses in educational sciences and subject matter didactics (e.g., mathematics education). These courses are also taught at university, stressing theory and critical reflection as opposed to being a mere how-to guide to methods and practices.

In this context arises a specific phenomenon with relevance to teacher education which is broadly discussed in German mathematics education research called the double discontinuity. The term *double discontinuity* denotes a situation where prospective teachers perceive a disconnectedness between the discourses of university mathematics and school mathematics, which they encounter on their way from school to university and back to school (see Winsløw, 2017; Hefendehl-Hebeker, 2013). The phenomenon is generally regarded as a problem of the educational system, as secondary school teachers who cannot make sense of the university discourse in a school setting are assumed to be less professional and less capable than those who can draw connections between school and university discourses. Its handling, however, is often located in the sphere of responsibility of university teaching. What Winsløw (2017) calls "*compartmentalisation of teacher education*" (p. 79) adds to this general impression of disconnectedness: Many German universities' teacher education curricula are organised in a way that promotes disconnectedness between the different subjects taught at university (i.e., subject 1, didactics of subject 1, subject 2, didactics of subject 2, educational sciences). This organisational separation mirrors differences between subjects that exist on the level of disciplinary cultures. In the following, we will subsume the *double discontinuity* and the *compartmentalisation of teacher education* under the term *phenomena of disconnectedness (of teacher education)*.

**14.2.1. The Course and the Concern for Reflective Agency.** This general context, as expressed above, is simultaneously the locus and the target of the teaching project on which we will now report: the creation and further development of a new mathematics education course for first year students.[7]

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[6] In German terms, our students are prospective teachers for the following school types: "Gymnasium," "Gesamtschule," "Berufsschule."

[7] In Germany, the federal states regulate the general structure of teacher training programs, which includes the distribution of credits among disciplinary, didactical and educational teaching units as well as the main content-related objectives. In Lower Saxony (the federal state Hannover belongs to), the regulations are specified in an Ordinance on Master's degrees for teaching professions

The course "Introduction to Mathematics Education"[8] was established in 2015 within an ongoing local reform project called *Leibniz-Prinzip*, which aims to improve teacher education at Hannover University by promoting reflective agency as the overarching educational goal of teacher education (see Dannemann et al., 2019). In view of the above-mentioned phenomena of disconnectedness, a major concern of our course development was and is to create connections between the mathematics taught in the first semester of university studies, the mathematics typically taught in school, and mathematics education theory and concepts. In the context of our research and development activities in the Leibniz-Prinzip project (see Khellaf et al., 2021), we formulated the following course goal, which represents our interpretation of the concept of reflective agency (see also Ruge et al., 2019), and which serves as guideline for course development and for the design of activities:

> In the first phase of teacher education, which takes place at university, explicit engagement with different discourses and views that are commonly present in institutions relevant to the teaching profession and with their justification strategies shall (be promoted and) lead to an enrichment of available perspectives on questions relevant to the teaching profession, foster reflection in students and ultimately enlarge their repertoire of possible responses to profession-specific situations. In addition to cognitive aspects the development of learning environments shall take into account affective-motivational aspects as well as the specific nature of scientific experience (Bachelard, 2002).[9]
> (Khellaf et al., 2021, translation by author)

The goal was formulated to be applicable to any teacher education subject – therefore it does not specify relevant discourses related to the respective subject-matter. In the case of our mathematics education course, relevant discourses are those of school mathematics, university mathematics, mathematics education (research), educational sciences and possibly other discourses present in society, which may involve for example common beliefs and everyday knowledge. This goal was furthermore created by and for teacher-researchers and phrased in a very general manner. It refers only very abstractly to student activities (e.g., "engagement") and doesn't yet define any concrete tasks. Therefore, it leaves a lot of leeway for the design of concrete tasks that aim to address students' prior knowledge and to foster students' development of interests and reflectivity. In order to facilitate task design based on this very general course goal, we also developed two principles of task design (Ruge et al., 2019) which suggest concrete ways of realising the course rationale by describing actions that should be promoted by inquiry tasks:

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(Nds.MasterVO-LehrM; `www.schure.de/20411/mastervo-lehr.htm`) that includes several annexes. These documents are based on agreements on teacher training made by the Standing Conference of the Ministers of Education and Cultural Affairs which all German federal states partake in (`www.kmk.org`). Innovative teaching interventions in teacher education that affect the compulsory programme in degree courses must stay within the boundaries of current regulations.

[8]The course consists of a weekly 45 minutes lecture plus 45 minutes of exercise class over the course of one year; its completion awards 4 ECTS. In 2020, the course has been completely digitalised and its structure became more flexible. In the first semester students are introduced to basics of didactic theory and practice text comprehension and academic communication. In the second semester the focus lies more on the comprehension of didactical questions and problems pertaining to specific pieces of mathematics and students engage in mathematical communication and the development of learning material.

[9]Bachelard makes a distinction between everyday life experience and scientific experience. Our interpretation of the concept builds on that distinction and acknowledges a difference between common knowledge and academic knowledge, as well as the cognitive and affective-motivational dimensions in relating common to academic knowledge.

(1) First design principle: understanding and comparing different perspectives and pieces of knowledge. Through the familiarisation with academic knowledge and theories and their comparison with everyday knowledge, students can encounter new perspectives and ways of thinking. In the process, cognitive conflicts can arise, which can motivate further investigations, and the direction of an ongoing investigation might need to be changed as the goal of the investigation is reformulated in accordance with the new insights. Tasks that promote the investigation and comparison of academic views have to be sufficiently rich and leave enough room for students to creatively engage with relevant perspectives.
(2) Second design principle: questioning one's own perspectives and knowledge. Typical problems and tasks are often strongly connected with typical ways of solving them, to the point where it becomes difficult to even imagine alternative possibilities and ways of acting. Actively imagining alternative scenarios with different possibilities can therefore cast light on current societal restrictions that may promote certain traditional approaches and ways of thinking. Such an activity can furthermore result in insights that motivate further investigation into societal restrictions. Tasks can promote such questioning of traditional views and habits by bearing strong resemblance to a typical scenario but then giving some incentive to reformulate the problem situation in different terms (than the usual ones).

In summary, the two design principles presented above aim to foster inquiry into different bodies of knowledge and their connections to each other. They inspired the creation of tasks whose solution requires switching between bodies of knowledge present in different but related discourses. One such inquiry task, that we use in our course, is "the graph sketching task," which aims to realise the second design principle.

**14.2.2. The Sample Task: Graph Sketching.** The task is introduced by a fictional school scenario (two pupils discussing an idea), in which mathematical questions are raised:

> An upper high school class reviews the topic of inflection point. One pupil draws on his desk neighbour's sheet Graph 1 below and comments: "Yo, I always wondered: If a function looks like this, does it have inflection points on the entire straight segment?"
>
> The desk neighbour, visibly amused, adds Graph 2 below and replies: "Look! Can't you do the same with a parabola? If you flatten it on the bottom, like this, wouldn't you also have lots of extrema? Infinitely many even!"

This introduction is followed by two graph sketches (see Figures 14.1 and 14.2), which in the fictional scenario were drawn by the two pupils:

(1) How many inflection points does Graph 1 in Figure 14.1 have?
(2) How many extrema does Graph 2 in Figure 14.2 have?

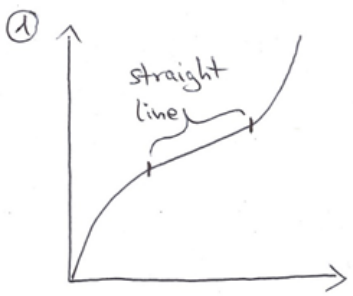


FIGURE 14.1. Graph 1.

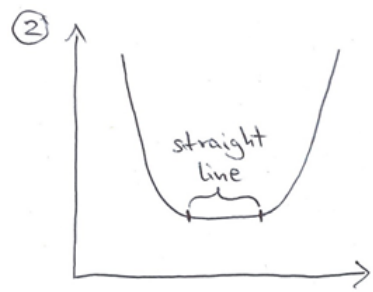


FIGURE 14.2. Graph 2.

The task given to our students consisted of a mathematical task (a) of giving a mathematically correct answer to questions (1) and (2) and a sufficient mathematical justification, and a teaching task (b) of proposing an adequate teacher response to these questions in a secondary school setting and specifying what didactical goals could be pursued in a discussion of these questions. We will limit our further discussion to the mathematical task (a).

The material[10] students are provided with in order to solve the task is a chapter on graph sketching from a German mathematics textbook for upper secondary school (Freudigmann et al., 2012, pp. 38–67). Although they are told that the schoolbook excerpt contains all necessary information to answer the mathematical questions, they are allowed to consult other sources if they like, such as lecture notes or other textbooks.

The textbook chapter we provide contains various types of information, such as exercises and examples, but students are expected to focus on the definitions and theorems of the chapter. Among the theorems are four which specify algorithms for finding extrema and inflection points on differentiable functions with specific properties (e.g., Figure 14.3: sufficient condition for the identification of extrema). These algorithms represent standard techniques to solve schoolbook exercises that ask to find these points of interest on functions that are typically given in algebraic form. The answers to questions 1) and 2), however, cannot be found through the application of these theorems: The theorems are formulated as unidirectional conditional statements "if A then B," where B postulates the existence of an extremum or inflection point; but in the cases of graph sketches 1 and 2, the sufficient condition A does not hold. The questions can instead be answered by looking at the definitions of extremum and inflection point: There are no inflection points on Graph 1 but infinitely many extrema on Graph 2.

**Theorem: Second sufficient condition for the identification of extrema**
Let the function $f$ be arbitrarily often differentiable on an interval $I = [a; b]$ and let $x_0 \in (a; b)$.
If $f'(x_0) = 0$ and $f''(x_0) < 0$, then $f$ has a local **maximum** $f(x_0)$ at $x_0$.
If $f'(x_0) = 0$ and $f''(x_0) > 0$, then $f$ has a local **minimum** $f(x_0)$ at $x_0$.

FIGURE 14.3. Example Theorem from Freudigmann et al. (2012, p. 52, translation by authors).

The presentation of the graph sketching task fulfils two purposes. Firstly, it caters to student demands for more practice-oriented activities in university teacher education: The fictional scenario is perceived as realistic in the sense that it might actually arise in school, and the task can presumably be solved within school mathematics as a schoolbook is the only material provided and declared to be sufficient for solving the task. Secondly, the presentation aims to lay the groundwork for the achievement of design principle 2 (p. 258) by providing sketches of graphs for which the standard solution procedure for this type of problem (i.e., checking necessary and sufficient criteria for extrema and inflections points according to the schoolbook theorems) does not work. The fact that the standard strategy for solving graph sketching tasks turns out to be unsuccessful and the possibly surprising task solution may give an incentive for reflection. Ideas about tangents of differentiable functions in the transition from

[10]All relevant definitions and theorems from the textbook are given in our IO3-materials on the PLATINUM website, `https://platinum.uia.no`.

school to university have been investigated in many ways, see for example Biza and Zachariades (2010) and the literature cited there. These studies focus in particular on the question of the relationship between ideas from geometry, for example in connection with tangents to a circle, and calculus, for example in connection with tangents as limit of secants. The task we have developed has other foci, for example, in that it aims at the relationship between procedural and conceptual knowledge of extreme value determinations, its different institutionalisations in schools and universities and, in particular, at issues of the didactic contract (Brousseau et al., 2020), that is, in this case, the adoption of responsibility for one's own mathematical actions.

**14.2.3. Experiences With Student Solutions and Reactions.** In arbitrary settings, the failure of standard strategies in itself will not necessarily provide sufficient motivation for reflection, as alternative solutions might be readily available and sufficiently plausible in the sense that they will not appear in any way noteworthy or problematic and will therefore not raise any further questions. In our specific case, however, thinking of the intended solution of checking the definitions proved difficult for our students (low solution rate, even in exam situations) and for many, the answers to mathematical questions 1) and 2) came as a surprise (we were told this in classroom discussions). The classroom experiences we made so far suggest that the graph sketching task can provide motivation for mathematical reflection on the significance and role of definitions in solving mathematical tasks or on the concepts of extremum and inflection point including aspects that are relevant for future teachers.

To give an example: Images of coastal roads and motorcycles are often used in German mathematics schoolbooks to illustrate the concept of inflection point. One imagines a mathematical curve to be a road on a map. While driving along this road, a motorcycle will lean to the left when the driver takes a left turn, and to the right when the road turns right. The point(s) at which the motorcycle is perfectly upright (perpendicular to the road's surface) while changing direction is said to be an inflection point of the curve/road. The fact that Graph 1 (p. 258) has no inflection points even though the motorcycle would be upright everywhere on the straight segment can motivate an investigation into the differences between Graph 1 and common school curves and give rise to discussions about didactic properties of commonly evoked imagery or about the nature of points in mathematics.

Furthermore, discussions about the reasons for the difficulties the students experienced can arise, which might lead up to a discussion of societal restrictions such as different didactic contracts (Brousseau et al., 2020) at school and university or differences between the mathematical discourses at school and university (e.g., emphasis of different mathematical techniques in teaching; strong focus on algorithmic procedures in typical teaching units on graph sketching in school). Such topics are not only relevant for the professional development of prospective mathematics teachers, but important for raising awareness of similarities and differences between the mathematics taught in school and practices of university mathematics.

Student difficulties that have the potential for such discussion and reflection include an initial avoidance of the intended difficulty of the task and subsequent mathematical discovery. Some students, for example, make the mathematical mistake of considering sufficient conditions to be necessary conditions as well (in logical terms, they derive $\neg A \implies \neg B$ from $A \implies B$), concluding from schoolbook theorems such as the one shown in Figure 14.3 that no extrema or inflection points can be found on Graphs 1 and 2. Other students undermine the didactic contract by arguing for an interpretation of the task instruction, that renders it solvable through an application of the schoolbook's theorems: They claim the pupils in the introductory scenario

must have made a mistake in assuming that their functions were really straight on the straight-looking sections, because the functions clearly have to be polynomials, and polynomials are never straight on open intervals (in this case the students also ignore the fact that constants are also polynomials). In both these cases of student difficulties (the mathematical mistake and the incorrect interpretation of the task instruction), the "artificially created" applicability of the standard theorems to the graph sketching task can motivate reflections and discussions about mathematics, about didactic contracts and about the differences between mathematical discourses in school and university. We will say more about students' handling of the graph sketching task in Section 14.4.

A last point we want to comment on is our idea that the task presentation (introductory scenario, schoolbook as material, graph sketching as topic) is successful in taking into account student demands for practice-oriented tasks in teacher education as affective-motivational aspects (see course goal in Section 14.2.1): We have met students who deemed the fictional scenario introducing the task plausible enough to become worried about their suitability for the teaching profession after experiencing the unexpected difficulty of the graph sketching task. This is noteworthy to us as our course has, in the past, met with repeated and at times fierce criticism by students who deemed its contents and tasks too theoretical, too far away from actual school practices and therefore irrelevant for prospective teachers ("a waste of time"). We will come back to this criticism in the next section, where we will reflect some of the contradictions and other relevant phenomena, we have encountered in our teaching project.

## 14.3. Phenomena and Contradictions of the Inquiry Teaching Project: Reflections Against the Background of Concepts Underlying Reflective Agency

The previous section concluded with the observation that students are at times quick to argue that the topic or the proposed activities of an assignment have nothing to do with school practice. Discussions of this point with students have in some instances become quite emotional, as students voiced indignation about having to work on some purportedly pointless task. Student calls for more practice-oriented course content are abundant in student evaluations of our course (though admittedly more so in older ones). From these experiences arises the question of how to reconcile conflicting visions (normative views) held by students and teachers of the desired learning outcomes and of the involved processes and activities in a teaching-learning situation. Some didactic choices, it appears, can lead to emotional reactions of resistance from the side of students although they might appear reasonable from the informed point of view of the teacher.

In the case of the graph sketching task, the graphs do not correspond to graphs of functions that are typically[11] taught at schools. From a didactic point of view, however, whether a mathematical problem might appear in school or not does not determine its relevance for teacher education. In our course, the graph sketching task illustrates that argumentation in school mathematics differs from that in university mathematics, and it illuminates specific differences between mathematical and teaching practices at school and university. This purpose is in line with our course goal (p. 256–258)

[11]The fact that Graph 1 and 2 (p. 258) are not typical for schoolbooks does not contradict the point made before that the fictional scenario is reasonably plausible. We will again point to the possibility of interpreting Graph 1 as a road and to the fact that the topic of extrema of constant functions arises in our schoolbook, albeit as a marginal mention.

which can in turn be further justified on the basis of psychological and didactical theories (Ruge et al., 2019). But how can these didactical considerations be relayed to students? As the reference to the work of Bachelard (2002) in our course goal indicates, we believe that it would be very difficult or even impossible for our first-year students to understand our course rationale. They have little knowledge of the specificities and practices of the teaching profession (they derive most of their impressions from what they have seen as pupils in their own school days) and are not yet familiar with the academic discourses underlying above didactic deliberations. This is why they will sometimes make demands that, from an informed perspective, seem counterproductive to successful[12] teacher education. But can or should students be ignored in didactical decision-making?

Despite the didactical considerations that support the graph sketching task, students' concerns cannot be ignored. Firstly, successful teaching depends on the acceptance of the teaching-learning-scenario by the learners (Rihm, 2006). In other words, didactical insights suggest that affective-motivational aspects should be considered in the creation of any teaching-learning-scenario. Secondly, there is institutional pressure to accommodate for student wishes to some extent: Student evaluations, for example, feed student opinions about our course back to the student council, who can then cut funds for our teaching staff if demands are not met. If evaluations are very bad, the faculty also has a response protocol with the intention to bring the evaluations up to an "acceptable" level. For the teacher this means that s/he has to manoeuvre between obtaining students' cooperation by catering to their expectations and articulated needs and insisting on certain didactical choices that appear necessary in order to be able to reach certain insights in the learning scenario.[13] In a broader view, a teacher in such a situation is dealing with an instance of the problem of theory and practice. This multi-faceted phenomenon has been broadly discussed in German educational sciences (e.g., Terhart, 2000) and is a recurring topic in discourses central to teacher education. The problem has to do with the way teacher education has been institutionalised within the German education system; it is connected but not identical with the phenomena of disconnectedness already mentioned in Section 14.2. The problem of theory and practice can be characterised as follows:

- on the level of didactic theory, certain philosophical frameworks insinuate a fundamental difference between theory and practice;
- on the level of implementation, a split between theory and practice is observable in typical institutional implementations of German teacher education in the following forms:
  - division of teacher education between two institutions (university and school/seminar) which are separated in terms of location and (institutional) structure, and
  - official division of responsibility for "academic/scientific education" (at university) and "practical education" (in schools / at seminars) between these institutions.

[12]Successful from the point of view of German institutions concerning teacher education and/or from our point of view as teachers.

[13]The fact that students do not necessarily react positively to teaching interventions such as proposed by IBME, or even reject them as demanding or even chaotic, is a thoroughly understandable reaction and must therefore be taken into consideration when designing interventions. Irrespective of this, student protest can lead to the termination of IBME-oriented teaching activities by the university administration, as described in the case study by our colleagues from Agder (see Chapter 11).

The phenomenon is typically associated with the disconnectedness between the (more "theoretical") discourses present at (German) universities and the (more "practice-oriented") discourses among teaching staff in schools (Schrittesser & Hofer, 2012, see Section 14.2: double discontinuity). A widely discussed problem that arises before this background and that affects practically all teacher education subjects is the above-described lack of tolerance for the theoretical nature of university teaching on the part of students, and motivational problems in connection with this (see for example Wenzl et al. (2018), for a commentary on this phenomenon). An orientation in favour of practice and against theory is also discussed in research in which dominant didactic currents or movements, such as competence orientation or an application orientation (in the sense of the modelling cycle), are not only problematised in an exemplifying manner with regard to their limits, but are themselves identified as expressions of insufficiently reflected institutional-societal contexts. Certain institutional-societal phenomena are addressed, for example, by Brousseau's notion of metadidactic shift (Brousseau, 2002, p. 261). Against this background, Gascón (2011) formulated the following critique of competence approaches: The shift leads to suggestions of teaching practices, in which the intention is to teach students problem solving, by trying to teach them how to learn problem solving by themselves (p. 36). Concerning the theoretical concept of 'competence' as it is proposed in educational sciences, it can be argued that curricular proposals in terms of competences (general, functional, technical, interpersonal, intellectual, etc.) actually turn the pedagogic problem into the solution: Making students acquire competencies is equated to teaching them competencies.

The other phenomenon mentioned before, of students undermining the didactic contract and avoiding dealing with the graph sketching task, is illustrative of another related phenomenon that we would like to draw attention to, namely the "problem" of tasks being used in a way not actually intended by teachers, especially those tasks that can be considered open and grant students a certain degree of freedom. This phenomenon is indicative of a fundamental principle in teaching-learning contexts: No teaching can force learning. Ultimately a triviality, this insight is recognised in principle by all learning and teaching theories. Conceptually, however, it is often relativised to some extent, especially in the way of not recognising the subjectivity of the learner and her/his agency. This happens particularly often in teaching and learning settings at typical educational institutions that are embedded in administrative structures, where the possibility of effectively planning, steering and controlling learning processes in order to move them in the intended direction is implicitly insinuated. In common traditional teaching-learning settings, learning efforts are often feigned or there is a reduction of deep learning to rote learning, mutually recognised by teachers and learners. Teaching without learning can naturally also occur in inquiry situations.

In the area of educational theory on inquiry teaching, in fact, we can find views that imply that the success of inquiry activities can and should be ensured by selecting tasks and managing classroom discussions particularly skilfully. What skilful means can be determined experimentally. This tendency in traditional approaches to inquiry teaching has led Holzkamp, among others, to formulate that such views are ultimately just particularly sophisticated attempts at manipulation with the aim of getting the learners to where the teacher, for whatever reason, wants them to be. They therefore merely represent a special variant of the otherwise widespread *teaching-learning short-circuit* (see Holzkamp, 1995). In response to such fundamental criticism of inquiry-approaches, Huck (2013) argues for the idea of inquiry-based teaching and learning by highlighting its conceptual focus on the importance of understanding

subject-specific connections and the relevance of the learner's own thoughts and use of their "practical" insight in learning a new topic. He does recognise the importance of guidance by a teacher but shifts the focus of attention to the fact that learning always includes the participation and engagement of the learner in the process offered by the teaching-learning activity. Letting learners make their own experiences and include their own insights in the teaching-learning activity stands in conflict with a one-way conceptualisation of teaching-learning.

This view is mirrored in our course goal (p. 256–258), which is centrally based on the subject-scientific-approach and its theory of learning by Holzkamp (1985, 1995): In our understanding of reflective agency we tried to conceptualise the promotion of actions or ways of thinking in our teaching as the creation of a space in which our students can enlarge their space of *action possibilities*.[14] By doing this, we strengthen the self-determination and agency of our students on a conceptual level and hope to consequently also achieve this in the realisation of teaching-learning scenarios. In this sense, we consider inquiry learning as an offer to "optimise" teaching-learning-scenarios in this direction, but it can no more force learning than other teaching concepts. In particular, there is no trick that guarantees that students take certain learning steps.

The issues just reflected on are also addressed in another strand of theory underlying our conceptualisation of reflective agency: *structural theory*. As example we can take the concept *autonomy-antinomy* (Helsper, 1996): Every teaching-learning relationship requires the recognition of a certain autonomy of the learner, since learning requires its own mental processes independent of the teacher. On the other hand, teachers in institutional teaching-learning relationships are required to ensure certain learning outcomes (see footnote 7). This antinomy is regarded, in structural theory, as constitutive of (institutionalised) teaching-learning relationships that cannot be bypassed.

In view of these theoretical reflections, we conclude regarding the previously described observations that we cannot avoid such student reactions, but rather have to understand them as possible and somewhat adequate expressions of the configuration addressed in the graph sketching task. In this sense, the avoidance of the task or the undermining of the didactic contract by the students should not be seen as a deficit but as a specific expression of agency that can be the starting point for reflections.

In the following, we want to relate our observations and their interpretations to our own development process (see Chapter 2 and 10). Regarding our professionalisation as teachers, we would conclude that it is too simplistic to value a teacher within IBME just based on the degree to which they (can) ensure that learners develop a practice-relevant, coherent and deep conceptual understanding of mathematics. Actually, if we would assess our own development process from this simplistic perspective, we would conclude that our efforts have been rather unsuccessful and that we are far from having achieved the goal of becoming successful IBME-teachers. But we would still claim a professional growth in PLATINUM: To us, the central point of all observations shared above is that the contradictions inherent in them are not resolvable just by us taking up an inquiry stance or optimising our inquiry activities and teaching practices (further and further), rather we have analysed and elaborated the contradictions for ourselves in order to be able to work within and upon these. One important aspect of professional growth within a complex setting such as ours is to come to terms with and accept

[14]The term 'action possibility' refers to an analytical category. The analytical categories of the subject-scientific approach "conceptualise the mediation between the vital necessities of sustaining the societal system as a whole and these necessities on the subjective level of the discrete individuals" (Holzkamp, 2013, p. 20).

the fact that many contradictions cannot be resolved, and partly lie far beyond the scope of teaching anyway. Accordingly, professional growth cannot consist in trying to resolve all contradictions, but in finding ways to come to terms with them—e.g., by locating, classifying and interpreting phenomena relevant to one's teaching practices. We have dealt with the contradictions by generating more information about them, by finding out what we can expect and by trying to move in this tension-filled field as smoothly as possible. We did not strive for a definite resolution, but are content that we can grapple with the contradictions in a more reassuring way.

So far, we have outlined the contradictory nature of our (institutional and societal) context in which our teaching practices are situated. In the next section, we discuss extensions and concretisations of PLATINUM concepts that allow us to integrate implications from the observations of this section with regard to our professional growth as researchers within PLATINUM.

## 14.4. Reflecting on Issues Regarding IBME, the Three-Layer Model and CoIs, and How They Underlie PLATINUM

Up to this point, we described phenomena as well as our interpretations of these with a focus on our goal of developing and establishing inquiry-based activities in our teaching. Now, we will reflect about our observations against the ideas formulated within the teaching-learning conceptualisation of IBME (14.4.1), the three-layer-model, and the conceptualisation of CoIs in PLATINUM (14.4.2). From our position as researchers within the global PLATINUM-group, we will also reflect on our experiences as teachers and researchers within the PLATINUM project, and we will argue for the need of a conceptual concretisations of the three-layer-model that accounts for societal and profession-related aspects, among others. These deliberations will lead us to a reformulation of the potentials the CoI-concept entails (14.4.3). To summarise and generalise: In this chapter we propose a deliberate approach to the constraints in the conceptualisation of IBME and indicate a restriction concerning its goals and an expansion of the concept of critical alignment.

**14.4.1. IBME as Counter-Horizon for Thinking About Teaching and Learning.** The conception of IBME (see Part 2 and 3 of the book) includes many statements about the kinds of learning activities that shall be elicited by teaching. Theoretical conceptualisations in which teaching practices are defined through their learning outcomes have been criticised as "short-circuit of the conceptualisation of teaching and learning" (see Holzkamp, 1995). They bring to attention only those kinds of learning activities and practices that are in alignment with pre-defined "learning goals" but leave little space for *critical* alignment within this narrow interpretative horizon of teaching and learning practices. In view of this, we ask: Is the notion of inquiry-based teaching and learning yet another expression of such a one-way conceptualisation in which teaching leads to learning? Or can we conceptualise IBME in a way that goes beyond such simplifications in its description of the relations between teaching and learning?

We argue that it is possible to take the idea of IBME as alternative outlook: IBME can offer a counter-horizon that opposes and challenges one-way conceptualisations of teaching-learning situations, as long as it is not regarded as a concretely achievable goal.[15] Thus, understanding IBME as a counter-horizon demands restraint in the

[15]To us, IBME is not a list of supposedly favourable "learning outcomes." To formulate goals of IBME in reference to a list of learning outcomes would be a step backwards towards a short-circuit of the conceptualisation of teaching and learning. So-called "learning goals" that are predefined by

setting of goals (in deviation from what is typically associated with IBME). In the following, we will explain in more detail in what way the notion of IBME holds the potential to oppose typical understandings of teaching and learning—we will rearticulate the potential of IBME.

In our institutional settings the typical or traditional horizon for interpreting phenomena in teaching-learning-situations are framed by a language of thinking in accomplished learning outcomes and "customer-satisfaction," which mingle with the ideal of fostering critical thinking. This horizon is rife with contradictions concerning the student as well as the teacher position: Students shall, on the one hand, align (in an uncritical manner) with pre-defined learning-outcomes and, on the other hand, be critical thinkers. Teachers are, on the one hand, considered to be autonomous in their teaching practices and committed to the subject-matter while, on the other hand, they are judged with respect to "customer-satisfaction" (which manifests in the questions of institutional evaluations and surfaces in students' wishes as well). In such a paradoxical framework, we are unable to express the relation between teaching and learning, between teachers and learners adequately. The counter-horizon IBME challenges aforementioned takes by offering a frame for inquiry into research and teaching practices that provides concepts to envision an extension of our possibilities of acting within and upon this paradoxical framing. Instead of limiting the understanding of the object to be studied—in our context, mathematics and mathematics education—to fixed learning outcomes and instead of subordinating teaching to "customer-satisfaction"-criteria, IBME takes into account the agency of both teachers and learners equally and articulates their ability and responsibility to engage in a critical manner with the subject-matter to be studied. Instead of restricting our understanding of teaching-learning-relations by pressing teachers and learners into predefined roles that limit their ability to engage with the object to be studied, the conceptualisation of this relation as a Community of Inquiry, in our interpretation, breaks with these narrow conceptions, in that it allows to ask for the learning opportunities an inquiry activity creates and the potentials we can create within and in trying to move beyond current restrictions.

To illustrate this point, we recall the above-mentioned phenomenon of students misinterpreting the instructions of the graph sketching task, thereby allowing themselves to apply standard criteria (see Section 14.2). The phenomenon sparked discussions in our LUH-group on how to deal with this situation. First, we need to acknowledge that the phenomenon took place in an inherently contradictory teaching-learning situation: On the one hand, the graph sketching task is designed to induce reflection and is open to further reflection. On the other hand, the task is embedded in an institutional setting in which, for the students, solving it is a matter of fulfilling external requirements, and in which, for us as teachers, it is tied to expectations that we plan our teaching in order to achieve predefined outcomes. From the standpoint of predefined learning outcomes those students clearly failed and their behaviour can be judged undesirable. Alternatively, the students' activities can (and should) be seen as a strategy to maintain or expand their agency: The "undesired" reinterpretation of assignments, for example, maintains agency by allowing students to deal with the task. The way they do it, of course, undermines the institutionalised didactic contract. Since we, in the position as representatives of the institution, cannot simply tolerate such reinterpretations, a conflict arises between us and the students. We can,

a teacher or the curriculum are actually teaching goals. A learner can, of course, formulate concrete goals for her or his learning process, and these can be closely connected to teaching goals. But if we strive for open inquiry, predefining learning outcomes actually run counter to this very ambition.

however, take up this conflict as a starting point for a joint reflection together with students on the inherently contradictory teaching-learning situation. In such a joint reflection, we would first of all acknowledge the students' compliance in delivering a solution. Secondly, we would recognise the fixating power of the school discourse, and the limitations it imposes: The simple solution of looking at the definition appears to be absent from a lot of students' space of possibilities. In this example case of the graph sketching task, our understanding of IBME in combination with insights into the didactic contract and knowledge of the various institutional positions enabled us to point to the potential for reflection on different mathematical discourses, on the nature of our teaching-learning-setting and beyond.

It should be noted that we understand these briefly described horizons and our following reflections not just as individual viewpoints that need to be changed or that shall be fostered. These horizons manifest in structural arrangements, narratives about teaching and learning and theoretical concepts. Therefore, the sustainable further development of teaching practices is not a simple matter of personal adaptation to, say, an inquiry stance in teaching practices or of becoming 'skilful' in selecting tasks and managing classroom discussions. It cannot be obtained, in fact, without altering structural arrangements as well as the conceptual understanding of teaching-learning phenomena present in teaching-learning-settings. Consequently, one goal of developmental research in this area can be the articulation of current restrictions in the form of structural arrangements or taken-for-granted perspectives within current theoretical conceptualisations. Developmental research should not be reduced to optimisation-concerns that limit its potential through the self-subordination to given restrictions.

**14.4.2. Reflections on the Relationship Between Professional Growth and Developmental Research.** The way the three-layer-model expresses the interrelation between theory and teaching development (see Section 14.1) differs from traditional models of developmental research that typically envision it as chronological four-step process consisting of research, followed by development, then design and, as last step, implementation (see also, Bauersfeld, 2000) arguing against this R-D-D-I model) and it goes beyond a dialogue between mathematics education researcher (on the one hand) and teacher (on the other hand) (Jaworski, 2004), in which researchers are perceived to be responsible for theory development and teachers are charged with the development of their professional (teaching) practice; the development of theory and professional practice, instead, constitutes a shared task. In our context in particular, theory development and development of teaching practice are strongly connected to each other. We see both aspects as being part of our professional growth with/in our group. How our practice as teachers motivates and guides our engagement in theory development was illustrated above, when we relayed our experiences with the graph sketching task. Theory development, for its part, can be seen as being part of our professional growth, because theories hold the potential of broadening our horizon of thinking about teaching-learning-relations. We would like to point out that we understand our involvement with the background theories of the PLATINUM project as critical alignment. In consequence, the critique we offer should not be understood as rejection, but as a critical questioning for developing the theory further in accordance with our experiences in our local context.

If we take a closer look at the three-layer model, the (further) development of theory in mathematics education (outer layer) and the further development of teaching practices (middle layer) are split and also separated by the objectives of inquiry (see Table 14.1, p. 254). We are going to have a closer look at the interrelatedness of these

two layers. In order to articulate the potential we see in further elaborating the theory behind the three-layer model and in strengthening the links between the middle and the outer layer, we need to make a theoretical excursion to explain our understanding of professional growth.

Inspired by the subject-scientific theory of learning (Holzkamp, 1995, 2013; Dreier, 1999; Ludwig, 2003) we conceive professional growth as extending one's own space of action possibilities in teaching-learning relations with/in a Community of Practice. This situates our further development within a Community of Inquiry (Jaworski, 2004). Within the subject-scientific approach, learners, teachers and researchers are perceived as "*producers of the life conditions to which they are simultaneously subject*" (Holzkamp, 2013, p. 20). The approach stresses the significance of these life conditions (specifically, teaching-learning conditions as well as the conditions of doing research in mathematics education) and it underlines the possibilities of the subject—learner, teacher, and researcher—to influence these life conditions in alliance with others. The analytical category *action possibilities* [Handlungsmöglichkeiten] refers to possibilities and hindrances to act in and on specific conditions from the standpoint of the subject.[16] Central to thinking in terms of action possibilities is the *twofold possibility* [doppelte Möglichkeit] to either reproduce restrictive conditions or to realise the possibility (however small[17]) of extending established practices and altering structural and socio-political conditions. This distinction is analytical and not to be mistaken for an either-or-relation. The introduced concepts can guide reflection processes about contradictory situations or persisting conflicts with regard to their structural and socio-political conditions. However, contradictory structural constellations are not generally assumed to be removable or resolvable. Structural and socio-political conditions are integrated into subjective reasoning in the form of societal-mediated meanings that constitute a person's space of action possibilities. The societal-mediated meanings that are grounded in these conditions constitute a space of available action possibilities. The space of action possibilities available to a subject is not fixed but can be extended. In consequence, we conceptualise professional growth as an extension of the space of action possibilities that is available to a professional.

The subject-scientific theory of learning emphasises the social dimension of this extension process. In alliance with others, it is possible to seize more opportunities for actions and participate in changing conditions that are constraining one's envisioned practices. Dreier (1999) relates this to community processes:

> the fundamental human duality between acting within the existing limits of social practice and extending its scope of possibilities is grounded in a similar duality of modes of participation [in a community], i.e. of participation in the reproduction of the current state of affairs or of contributing to change it so that participants may extend their degree of disposal over the social practice. (p. 6)

We regard the notion of CoI as a sociocultural construct (Goodchild, 2014) which, as a framework, accounts for such activities that tackle shared socio-political conditions.[18] Jaworski (2004, 2006) points out the risk that community processes could

[16]Action possibilities include, by definition, both opportunities and constraints.

[17]This follows the basic assumption that in antagonistic class conditions, the attempt to gain more control over conditions is always accompanied by the risk of getting in conflict with the agents of power and provoking restrictions.

[18]We share the conviction that research in mathematics education needs to integrate social theory and cannot disregard broader societal conditions in the interpretation of phenomena that can be found in teaching-learning. Otherwise it would conceal the socio-political dimension of mathematics education by reducing didactics to the development and implementation of teaching strategies. To adequately capture these phenomena, it is important for us to be equipped with a theory that provides

hinder further development with/in a community: An unquestioned alignment with and participation in the practices of the local community could lead to a reproduction of undesirable practices. She therefore emphasizes the importance of a critical alignment with teaching-learning practices with/in a community. For us, such an inquiry stance towards one's own practices includes inquiry into learning, teaching and research. Our reflections and debates in the LUH-group can, in this context, be understood as supporting an ongoing *(self-)understanding process* that takes place between the community members and the socio-political conditions in which the professional work is situated. This includes the "reflection on social requirements and conditions in an attempt to (re)establish self-understanding in individual situations of action and to be able to act in a competent [/professional] manner" (Ludwig, 2003, p. 1, translation by author). Therefore, "seeking (self-)understanding" denotes the attempt to gain knowledge about and to trace one's own personal and structural entanglement in contradictory situations, which can consist in the (unwitting) participation in community practices which run counter to one's own interests and desired practices. By striving for (self-)understanding, we attempt to gain more disposal over our research and teaching practices. Rihm (2006) points out that in our routinised daily work, we often 'interpret' situations within the horizon of the typical space of action possibilities of our daily practices. This means we unquestioningly accept quite a number of aspects of typical ways of working in our community. 'To understand,'[19] on the other hand, means to gain knowledge about and to trace one's own structural entanglements in contradictory situations (and possibly to reconstruct participation in community practices that are contrary to one's own interests). Seeking understanding, therefore, means to widen one's own view and to transcend the horizon of our everyday entanglement. By calling into question one's own reasoning, understanding goes beyond a reflection of current conditions as parameters that set boundaries for the exploration of (the range of available) options. It entails questioning one's own interpretations of phenomena related to teaching and learning.

Seeking understanding to gain more disposal over our researching and teaching practices is what we understand as critical alignment within our group. This kind of (self-)understanding goes beyond a merely introspective and individual way of progressing (Rihm, 2006): The intertwining of our perspectives allows us to take a metastandpoint that makes it possible to recognise the interrelation of different practices prevalent in society and the group (different research, teaching, and learning practices). This reflexive distance does not only allow us to identify supporting and obstructive conditions and to question our own interpretations, but also to recognise potentials of altering conditions (Häcker & Rihm, 2005, p. 375).

Within the LUH-group, we cooperatively try to widen our viewpoints and do not distinguish between researcher and teacher as fixed positions. Regarding the middle and outer layer of the three-layer-model, viewpoints on how teaching and learning are related to each other are important issues for developmental research and teaching practice alike. Our reflections within the LUH-group are of importance for our professional growth as teachers as well as researchers and cannot be assigned to one specific layer.

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a language for characterising human actions within social conditions. For this purpose we rely on the subject-scientific approach.

[19] 'Interpreting' is then not opposed to 'understanding.' Rather, 'understanding' simultaneously suspends 'interpreting' in itself and transcends it (Holzkamp, 1985, p. 395).

**14.4.3. Reformulation of Potential of CoI.** To us, an inquiry stance towards teaching and learning means thinking in alternatives and potentials.[20] In consequence, we do not take current conditions and approaches to teaching-learning mathematics (education) for granted, but scrutinise them for obstructive elements and possibilities to think beyond the narrow horizon of current practices. This objective can be related to an emancipatory objective of academic work that is also of key importance for building up a professional knowledge base for teaching (Langemeyer, 2020). In teaching-learning relations, we often act in a restricted manner, in a modality of alignment with or subjection to given obstructive structures.[21] But education can also be thought of as a cooperative activity directed towards extending each participant's space of action possibilities, which also includes extending each participant's control over restrictive teaching-learning conditions. It entails the possibility of overcoming obstructions to teaching and learning in alliance with others. Research can provide concepts for reflection, concepts that promote the process of seeking self-understanding for the professional task of teaching. Linking research activity in mathematics education and the teaching and learning of mathematics (education) with each other within a Community of Inquiry has the potential of developing and building on theory that integrates several standpoints of the teaching-learning relations. These standpoints are anchor points for the reflective task of decentring from one´s own viewpoint and jointly developing a meta-standpoint. The process of decentring can be described as a combination of zooming out and zooming in (Busch-Jensen & Schraube, 2019).

The strength of a conceptualisation that locates CoI on all layers of the three-layer-model (inner, middle and outer layer) lies in its ability to draw attention to the possibility of engagement in terms of a critical alignment, that calls for inquiry into the subject-matter as well as inquiry into conditions that obstruct teaching-learning processes and, thus, also restrict inquiry into the subject-matter.

## 14.5. Concluding Remark

The presented reflections can be understood as our critical alignment with PLATINUM concepts of IBME and CoI, their potentials and limitations. Our reflection resulted in a further development of theory (expansion and differentiation) that is based on our experiences as teachers and researchers in the LUH-group, our participation as researchers in the PLATINUM project, and our theoretical stance towards mathematics education. We have presented our reflections in this contribution from the perspectives of two different positions: teacher and researcher. These two positions are of course not independent of each other, since that would in our context imply a "splitting of our identity." Rather, the two positions are dialectically connected. Unfolding their nonlinear relationship in a linear text was a great challenge for us and led us to make an analytical distinction between issues that we considered to be more of relevance for the position of a teacher in a developmental research project and issues that we thought to be more relevant for the position of a researcher entangled in the practices s/he is inquiring into. The present contribution documents what remains an ongoing discussion of how to grasp and categorise these issues.

We regard our puzzlement concerning theoretical approaches and making sense of them as relevant personal experiences within PLATINUM. To us, working with and developing theory further is not only a cognitive task, but also involves affective-motivational aspects. We acknowledge both the cognitive and affective-motivational

[20]Even if these are not yet realisable under the given conditions.

[21]Preservation of the status quo, or safekeeping one's own position at the cost of the (re-) production of restrictive conditions.

facets and their relatedness in our practice of theory development. In alliance with each other, we take our personal experiences and sensitivities in teaching practice and theoretical work as a starting point for further development. This entails supporting each other if one struggles with opposition to her or his teaching practices or theoretical stance and discussing and classifying doubts. The emotional support of the group is essential, but to work in alliance with each other, to us, necessarily involves a deliberate decentring from one's own viewpoint.

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