Inquiry in University Mathematics Teaching and Learning: The PLATINUM Project
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CHAPTER 13

The First Experience With IBME at Masaryk University, Brno

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13.1. Introduction

This study provides an insight into the emerging community of inquiry at Masaryk University in Brno, Czech Republic. First, we present the historical and institutional background that determines the conditions in which our teaching team operates. We briefly summarise the possible ways of professional development intended for academics at our university (except for pedagogy students preparing for the future career of primary or secondary school teachers). We believe that the relatively low institutional support of academics in this field dramatically influences the limited professional knowledge of didactics and pedagogy among our team members. Furthermore, we describe our community, which is characterised by considerable fragmentation—both in terms of courses taught and belonging to individual faculties and university workplaces. The physical distance between the buildings where our teachers work is also one of the limiting factors for the functioning of our community. Due to this fragmentation, it is also difficult to characterise the functioning of the community as a whole. Individual sub-teams differ in their approach to designing and implementing IBME tasks in teaching, monitoring, and evaluation. Therefore, we present three different examples of approaches to IBME education, which in our opinion, demonstrate this diversity. Their analysis is presented in three separate sections.

13.2. The Historical and Institutional Background

In this section we picture the pedagogical education of university teachers in the Czech republic and the professional development of lecturers at Masaryk University.

13.2.1. Pedagogical Education of University Teachers in the Czech Republic. While young academics are well prepared for a scientific career during their studies in the Czech Republic, they are usually not prepared for a teaching career at all (Čejková, 2017). Traditional universities combine science and research with teaching, but the attention of academic staff is focused more on science and research at the expense of teaching. This is most often explained by the fact that research performance is the subject of continuous evaluation; it is tied to both the financial remuneration of academics and their career growth, while a system for monitoring the quality of teaching is not present at universities. However, as mentioned by many authors, excellence in research does not necessarily lead to excellence in teaching.

The topic of teachers' professional beginnings is widely addressed in Czech pedagogy, but almost exclusively in the context of primary and secondary education

(Šed'ová et al., 2016). Historical roots of this situation in the Czech Republic are uncovered by Vašutová (2005): the system of pedagogical education of university teachers nearly disappeared in the 1990s. In the post-revolutionary period, we began to rely heavily on a personal approach to teaching and individual responsibility based on renewed academic freedoms. However, due to new requirements and direct and indirect pressures on higher education institutions, the scope of activities of university teachers has demonstrably expanded, performance started to require more time, and its complexity increased. So a new generation of university teachers is not well prepared for their university career (Vašutová, 2005), mainly because they were educated only in the field they now teach and lack teacher education. This shortcoming in undergraduate training is not compensated even when novice lecturers start teaching; there is usually no systematic work with PhD students and novice academic staff.

In the absence of institutional support mechanisms, some academics work independently on their professional development in teaching, even though they do not receive many external incentives to improve their teaching performance. As mentioned by Čejková (2017), insufficient pedagogical education of beginning university teachers is most often replaced by the use of personal contacts. This is realised in various ways—from direct requests for advice through informal interviews and sharing of resources to the observation of colleagues, and so on (Pataraia et al., 2015). Although learning from colleagues is an important socialising element, some authors point out possible risks. Hativa et al. (2001) argue that an unplanned and uncontrolled socialisation process can lead to the acquisition of fragmented pedagogical knowledge and unsubstantiated assumptions about what teaching practices are desirable and effective. Observational learning of beginning teachers can lead to an undesirable effect in following inappropriately chosen patterns. In addition to using the services of more experienced colleagues, beginning university teachers use another source of knowledge, which is similarly common and similarly risky, namely their own experience as student. This is another crucial source playing a key role in the preparation and implementation of teaching (Oleson & Hora, 2013). Similarly, beginning teachers acquire professional skills by trial and error in their own teaching (Hativa et al., 2001). The is no assurance whether they achieve the required quality in this way.

13.2.2. Professional Development of Lecturers at Masaryk University. The problems described in the previous section are also present at Masaryk University (MU). The first systematic attempt at MU to educate new lecturers in teaching competencies is relatively recent; in 2017 the Pedagogical Competence Development Centre (CERPEK) was established within a framework of a local project. It aims at increasing the level of pedagogical competencies of beginning university teachers (having less than 5 years of practice). CERPEK offers a two-semester study programme "Development of Pedagogical Competences," which consist of four parts:

- The laboratory of pedagogical competences (covering topics such as quality of university education, preparation for teaching, communication, students' evaluation, feedback reflection, use of modern technologies, etc.);
- video-reflection on recorded teaching (adjusted to participants needs);
- teaching workshops;
- the mentor programme.

Participants of this programme first attend a twenty-hour intensive seminar, then they learn through online courses. During the year, they have to work out various tasks, including preparing a model lesson to be recorded. As a result, they will receive feedback from course instructors. The programme should help participants identify their strengths and weaknesses. Attention is paid to various topics—how to plan teaching well, how to effectively pass information, what tools to use, etc. However, one member of our group participated in the course and he described the experience as not very beneficial. In his view, the workshops have been too general and the individual topics were not integrated into a unifying framework.

The programme is prepared by a team of specialists in the area of pedagogy, andragogy, and psychology as a general didactics course without subject specificity, so it does not focus at all on teaching mathematics. The capacity of the course is quite limited—about 30 places are offered each year. Up to this date, less than 100 participants graduated in this program (which is a very small fraction of a total more than 2600 teachers employed at the university), only seven of them were from the Faculty of Economics and Administration and only one was teaching mathematics. This situation may change in the future if the centre manages to raise additional funds to expand its offer. CERPEK offers no support for inquiry-based teaching and learning yet. We tried to establish collaboration between the Centre and our CoI, but the COVID-19 pandemic complicated the situation. In 2019, some arrangements were done to prepare together the organisation of the Workshop on Inquiry-Based Education. PLATINUM community experts were asked to hold the workshop for the CERPEK Centre, but it had to be postponed until the epidemic situation improves.

Until CERPEK is able to provide sufficient systematic support to beginning university teachers, some activities organised from the bottom, by young academics themselves may be of great benefit. For example, the youngest member of our CoI greatly appreciated participation in the course “DUCIT Teaching Lab” primarily aiming at future lecturers of the Faculty of Informatics. If the course is not full, young lecturers of mathematics (mainly doctoral students) from other faculties may enrol in this course. According to our colleague, the course helped him to answer important how-to questions such as “How to structure the lesson,” “How to motivate students,” “How to ask properly,” and “How to give and get feedback,” but also questions connected directly to mathematics teaching.

There are no such activities organised for older teachers, but the motivated ones seek other possibilities to enhance their teaching skills. For example, we can mention several two-day courses for academics held by the university language centre such as How to Start Your Term Effectively, Communication with Students, Feedback and Evaluation, Intercultural Teaching, or five-days summer school Academic Skills in English. However, there are no official courses or university programs focused on inquiry-based pedagogy.

13.3. The Community of Inquiry at Masaryk University

In this section we introduce and characterise the Community of Inquiry (CoI) at Masaryk University, describe how this community works, and present the first steps of the MU CoI in introducing inquiry-based teaching and learning in some courses.

13.3.1. Characteristics of the MU CoI. Our Community of Inquiry was established in 2018 in connection with the participation in the PLATINUM project. We are all lecturers; there are no didacticians or researchers in mathematics education among us. The group is made up of around ten people, but this is changing over time—some members left the university, but new people joined the group. The majority of CoI members are from the Faculty of Economics and Administration (some are teaching statistics courses, and others are teaching mathematics courses). Two members of our CoI are also teaching the course Mathematical Analysis 1 at the Faculty of Education, and one of them works at the Support Centre for Students with Special Needs (Teiresias Centre) as well.

Team members involved in the cases described in this chapter form a group of two experienced lecturers (Mary and Marge) with more than twenty years of practice, three teachers with an intermediate experience (Lenny, Patricia, and Luke) working as course instructors, two post-docs (Hanna and David) involved as observers in the lessons, and one PhD Student (Tamara). The names of the participants are pseudonyms so that their identity is not revealed since we publish their opinions using excerpts from their reflective narratives.

Previously, most of us did not even know the ideas of Inquiry-Based Mathematics Education (IBME), but gradually within the project, we got acquainted with the IBME principles and, as far as possible, started to include them in teaching. Some of us inclined to these principles intuitively before participating in the project but lacked the theoretical support, as illustrated by perceptions of two teachers:

> It took me time to understand the term 'inquiry.' However, though I didn't know this term, I applied at least some principles of inquiry in my teaching since I started to teach. I didn't know the term 'procedural teaching,' but I always struggled to suppress teaching algorithms in favour of understanding. However, still, I need to move from students' understanding based on my explanation of the underlying ideas to their investigation based on thinking-provoking tasks. This is for me one of the main objectives of the PLATINUM project. I wish to collect either existing or develop new inquiry-provoking tasks, which could be implemented into our statistics courses. As we are rebuilding the course of Statistics 1, we have an opportunity to incorporate IBME principles systematically. For that reason, I appreciate any source of IBME tasks for probability and statistics, or possibly links to any developed curriculum with IBME element. (extraction from the reflective narrative of Mary, June 2019)

> I have several experiences with tutoring students on every level of education, from primary school to university. I always tried to lead them to solve mathematical problems on their own. I only push them the right way by questions and showing similarities to already-figured problems. But I could never imagine how to use this approach in classes with more students and with such a full schedule. So I am very happy for this project and for the opportunity to be part of it. I consider it very inspiring to get ideas on how to provide this way of teaching from so many people with the same interest. (extraction from the reflective narrative of Tamara, March 2020)

> I had supposed that there would be many courses focused on teaching students how to teach. Still, there were only a few such courses, and these were newly established. On the other hand, it was at least some progress in attitude to teaching. In these courses, I first met with Inquiry-based tasks, using GeoGebra and an interactive blackboard. During my PhD study, I participated in the conference STAKAN, focused on teaching statistics; some contributions also involved inquiry-based principles. This occasion motivated me to try to implement some of the activating elements into lectures from the beginning of my teaching career. (extraction from the reflective narrative of Patricia, March 2020)

13.3.2. The CoI Meetings and Discussions. Before we engaged in the PLATINUM project, the regular cycle of practice evolution applied in our teaching included the following steps: plan for teaching, act in the classroom, reflect on experience, feedback to regular planning. Now it was transformed into a teaching inquiry cycle by introducing systematic observation, analysis, and reporting. As described by Goodchild et al. (2013): "in the inquiry cycle, systematic observation and analysis inform the reflective process which provides possibilities for re-planning in better informed ways leading to a more knowledgeable design of teaching" (p. 398).

Moreover, the level of cooperation and communication between the teachers has grown significantly.

Our group of inquiry is rather heterogeneous; its members come from several faculties, operate in various workplaces, and teach many different subjects. This makes the coordination of the group activities somewhat tricky. We meet on an irregular basis as a whole group, sometimes together with the people from the Brno University of Technology community (e.g., during the workshop Teaching Introductory Statistics with Technology in February 2019 or during the workshop with Barbara Jaworski and Simon Goodchild in December 2019). As we do not have a strong background in the theory of mathematics education, we try to use some scientific resources and the broader PLATINUM community's support. The frequency of meetings of the smaller groups connected to a specific course is at least once a month, but short informal discussions happen more often, during lunches, via email or social media, phone calls, etc. The meetings last mostly about one hour; they are usually unstructured—we discuss our experience from tutorials, topics for future seminars, and assessment tasks. The frequency and intensity of the group meetings involved in statistical courses has increased lately because these courses are being redesigned completely.

13.3.3. Implementing IBME in Some of Our Courses. Inquiry-based activities were introduced in the courses Statistics 1, Mathematical Analysis 1, and Mathematics 2. All courses are taught through lectures (2 hours per week) and seminars (2 hours per week).

- The course *Statistics 1* is obligatory for all bachelor's programmes at the Faculty of Economics and Administration. It is a large course with about 450 enrolled students divided into 20-25 seminar groups taught by 8-10 instructors each year.
- The course *Mathematical Analysis 1* is taught at the Faculty of Education of Masaryk University. There are three seminar groups, each of which have 20-25 participants, so in total 70 students of the bachelor's programme Mathematics for Education. Most of the students are in the 2nd semester of their studies. After they successfully finish their bachelor studies (3 years) and the following master studies (2 years), they could start their professional career as mathematics teachers in secondary schools.
- *Mathematics 2* is an obligatory course for first-year students in the Master program of Economy, Finance, and Management. There are about 90 students enrolled in the course each year. The course syllabus covers selected topics from calculus and linear algebra, such as constrained optimisation (especially linear programming) and differential equations.

These courses represent three different cases of IBME application ranging from including short inquiry-based tasks to each seminar (Statistics 1), over team project assignments in Mathematics 2, to whole IBME teaching units in Mathematical Analysis 1. The goals for which we use IBME tasks in individual courses differ: in Statistics 1 and Mathematics 2, we usually aim at introducing new concepts and arousing interest in the topic under discussion; whereas the goal in Mathematical Analysis 1 is usually the repetition of basic concepts and ideas, and deepening of students' understanding. Activities in Mathematics 2 are focused on applications, and they are intended to bridge theory and practice. The ways of monitoring and evaluating the education process also varies: observations by other teachers were applied in the first two courses, whereas in Mathematics 2, the teacher reflected on student presentations and reports from a feedback questionnaire. While observers made a structured record of what was happening in class in Statistics 1, observations from teaching were passed on orally to the teacher by the observer in Mathematical Analysis 1. An example of feedback and the subsequent reflection of teachers can be found in the following sections. We start with a newly designed Statistics 1 course. Next, Luke and his colleague Lenny report on their experience as instructors of the seminars in the Mathematical Analysis 1 course, and finally reflections from Mathematics 2 are summarised.

13.4. Statistics 1: The Experience of Tamara and Patricia

During the planned revision of the course, almost two dozen IBME teaching activities were created. Some of them are based only on a moderated discussion of students (or with the use of aids like dice, lottery equipment, or multi-coloured cards), others need the use of software, for example, to solve assigned tasks using simulations, etc. Three examples of these short activities are shown in Figure 13.1

IB tasks for Statistics course, 1st semester

Exercise 1 (Flashcards [**week 1**]). *We have 3 larger flashcards (nominal, ordinal and quantitative) data and various examples of data on smaller flashcards. Students are divided into smaller groups and have the task to assign variables to the correct data type. Evaluation follows*

Instructions. *Teachers get cards sorted according to the data type. Some may be controversial, depending on how it is taken - specifically, e.g. US presidents - we can sort them in time, but sometimes we look at them as nominal - depending on the context. Discuss with the students.*

Exercise 2 (Questionnaire [**week 2**]). *The teacher will bring the printed questionnaire to the lesson in sufficient amount and let the students to fill it. Then, instead of collecting it, teacher announces that the answers do not matter, and say that he/she is interested in the opinion on the questionnaire as a tool of data collection. Whether the questionnaire looks professional, questions are formulated correctly, etc.*

Instructions. *The questionnaire is of course all wrong. The mistakes were made intentionally: missing some motivation, explaining why they should answer, the overall composition of the questions is meaningless, etc. Discuss.*

Exercise 3 (Random generator [**week 3**]). *Work in pairs. Select* **random** *ten people from the list of people. Use a 10-sided dice to generate random values.*

Think about whether your chosen people selection process is really random, i.e. whether each person has the same probability of being selected.

Then randomly select 10 men (women) from the list.

Instructions. *Students in pairs have a randomly sized sample of 10 from the list of people. They have a 10-sided cube. Then the same task, but with limitation (limited to men, etc.).*

Activity follows with a discussion of how they have proceeded and whether their process is truly correct.

FIGURE 13.1. Activities for the course Statistics 1.

All tasks were originally prepared by one of the teachers and their preparations were no subjects for debate within our CoI (only after the realisation of the activities). To evaluate the benefits of the inquiry-based tasks from the students' perspective, we asked them to fill out the feedback form after each IBME unit. We also included questions focusing on achieving learning objectives and questions associated with students' engagement. The evaluation process is at the time of writing not yet finished.

Referring to the three-layer inquiry model described in Chapter 2 (see also Jaworski, 2019), the connection to higher layers of inquiry was realised through observations of lectures and seminars. Two colleagues, in the role of observers, were taking minutes to capture the structure of lessons and timing of the tasks. An example of an observation report on the activity in week 2 captured by Tamara is shown in Figure 13.2. Before the COVID-19 pandemic, they were present in classrooms so it was possible to add comments on students' engagement and their inquiry development. After switching to distance learning, lessons were taught via distance learning platforms. Some of them were recorded, so it is possible to reflect on them even without an observers' presence.

Minutes	**Observer's comments**
14:00 - Lesson started with introduction to main topics of the lesson: random selection, data collection.	
14:02 Starting the IBE activity. The teacher gave students a task to fill in a questionnaire.	
14:08 - The teacher announced a change of assignment. Students had no longer answer the questions, but they should decide if the questions were correctly formulated or how to make them right.	It was easier for students to find errors and reword particular questions after answering the questionnaire. Students were more open to share their opinions. As usual, there were a couple of very active students, that answer often and some with any response. Students were very creative and had various suggestions.
14:13 – Teacher read each question aloud and opened the discussion. After discussion to every question from questionnaire teacher summarized the proposed solutions and joined own perception. There where an example of a manipulative question, incorrectly chosen intervals for age, scale without explanation of extremes or double-barreled question, etc.	
14:34 – The teacher explained why they did this activity and gave some recommendations on how to make a questionnaire right.	

FIGURE 13.2. Observation report from Statistics 1: activity on data collection.

The observations followed by discussions within the group of course tutors revealed that two or three of the activities missed their goal and would be disposed of. Though, most of them were found useful and would be kept for the future use (some after slight adjustments of formulations in task assignment for better understanding). The teachers agreed that the lessons with IBME units are more demanding than traditional ones, but most of them welcome the challenge. Especially the younger of us say it is more fun to teach this way:

> I believe that inquiry-based tasks can help make students be more active, be involved in seminars and better understand the discussed topic. Many students are afraid of maths and stats courses, and they need to be encouraged, and inquiry-based tasks can help them to get more confidence. On the other hand, these tasks are usually time-consuming. Therefore a good option for me is to involve only short inquiry-based tasks

to each seminar. I think students were more communicative and able to create their ideas (extraction from the reflective narrative of Patricia).

But not all tutors are in favour of the activities. Some of the tasks are perceived as problematic, especially those not having a unique solution. Teachers also report that sometimes it is not possible to keep discussion under control; so it gets too broad or off-topic. That may be limiting students who need a more systematic approach to learning and confusing students with weak foundations of underlining theory. One of the tutors even declared the intention to quit teaching this particular course and the mentioned issues may add to the reasons for his decision.

13.5. Mathematical Analysis 1: The Experience of Lenny and Luke

Mathematical analysis 1 is a course offered to students at the Faculty of Education. Its main topic is differential calculus: real functions of one variable (7 weeks) and real functions of two or more variables (3-4 weeks). Students of the course attend lectures (2 hours per week led by the lecturer who is not a member of the CoI) and seminars (2 hours per week led by us—Lenny and Luke, the main authors of this section).

Students are introduced with topics during lectures on a more theoretical level. During seminars, we (Lenny and Luke) should continue, add more practical information and guide students to understand these topics. The content of sessions is chosen by us, not by students, and given by a strict curriculum of the course. We usually prepare one document for each session including the most important facts, examples to compute during instruction or at home, and the recommended resources to read. This file is shared with students before the session starts. During sessions, we discuss the key terms and try to activate students to come with their answers, comments, questions. We also guide them during practical solution of examples and give them feedback. Students are supposed to solve the problems by themselves or in groups, not only wait for explanation of an instructor.

The lecturer is very close to retirement, and we decided that we would not ask him to modify his lectures to be more inquiry-based. But he agreed we can change seminars and add inquiry-based tasks or units. We started to plan this modification at the beginning of January 2019, so we didn't have more than two months for the design and development phase as we planned to use the first IBME tasks at the beginning of March. That's why we focused on particular topics and didn't modify all seminars' sessions, but only three of them.

We prepared two smaller IBME tasks[1] and one IBME unit based on two worksheets[2]. We designed IBME activities to enable students to:

- understand key theoretical terms such as the limit and the derivative of a real function of a real variable,
- work with applications for plotting graphs of 2D functions (GeoGebra, Wolfram Alpha, etc.) in order to interpret the above mentioned fundamental concepts of the course geometrically, and to
- understand the relationship between the first derivative and monotonicity, and the second derivative and convexity/concavity.

All IBME activities were organised in the same way, going from a rather closed format to an open format. Students got detailed instructions and were asked to do small consecutive subtasks within groups of 2–4 members. They were encouraged to use all reachable resources and work with applications for plotting graphs of 2D

[1]We selected one to be described in Section 13.5.1.

[2]The IBME unit is discussed in more detail in Section 13.5.2.

functions on their cell-phones, tablets or laptops. These teams worked separately, we observed their activities and help them individually in case they got stuck or didn't find the answer to the particular question they posed. After they finished, we started the overall discussion and asked the students to come with remarks and findings. This part was very open: we only moderated the discussion and tried to activate students to response and give their own informal explanations. Finally we invited students to summarise and formulate conclusions most relevant to the topic.

13.5.1. IBME Task—Limit of a Function.

The task instructions: Make groups of 2-4 people. One of the group specifies limit conditions or requirements on continuity of an unknown function. The others try to find an example of a function which meets the requirements. You can change the roles then. Examples of requirements:

(a) Find a function $f(x)$ such that $\lim_{x\to 3} f(x) = 5$.
(b) Find a function $f(x)$ such that $\lim_{x\to 3} f(x) = 5$, but $f(x)$ is not continuous at $x = 3$.
(c) Find a function $f(x)$ such that $\lim_{x\to 0} f(x) = -\infty$.

Goals: We designed the task to enable students to repeat the knowledge that they had received during the previous theoretical lecture. We intended to give students the opportunity to

- recall the concept of limit and continuity of a real function, and
- get back to their own resources and read through the notes they made about the topic during the previous lecture.

Because the task was designed as a game between students, we expected them to be motivated enough to come with requirements that would not be easy to meet, for example limits at infinity, infinite limits, or limits at the point of discontinuity. When giving a solution, students were supposed to justify their proposal and it was open in which way they did so—whether they explained it verbally, computed that or demonstrated it on the function graph drawn by themselves on a sheet of paper or plotted by GeoGebra, Wolfram Alpha, etc.

Tutors' experience: Our impression was very positive. All students made groups and started actively working on the task. We recognised it was fun for them. More than half of the groups used the examples we had offered together with instructions. That was not our intention, so we agreed to ask students explicitly to create their own tasks next time when we repeat the session. In the end, we invited students to come with their own cases. It was useful because they made several mistakes and we all started reasoning and explaining all kinds of limits of functions.

13.5.2. IBME Unit—Monotonicity, Convexity/Concavity.

Unit scenario: In this case, we prepared two worksheets and planned the whole 2 hours' session (100 minutes) as follows:

(1) Working on the 1st worksheet on monotonicity of a function, see Figure 13.3 (25–30 minutes),
(2) Practical solution of other examples selected by teachers[3] (20 minutes);
(3) Working on the 2nd worksheet on convexity/concavity of a function, see the Figure 13.4 (25–30 minutes);
(4) Practical solution of other examples selected by teachers[4] (20 minutes).

[3]Students were asked to specify intervals of monotonicity and local extrema of selected functions.

[4]Students were asked to specify intervals of convexity/concavity and inflection points of selected functions.

Worksheet – Monotonicity of a function

Let's consider the function $f(x) = \frac{1}{4}x^4 - x^3 - 2x^2 + 3$.

(a) Make the graph of the function using GeoGebra.

(b) Specify intervals on which the function increases or decreases; write it down in the following table:

	$(-\infty,\ -1\rangle$	$\langle -1,\ 0\rangle$	$\langle 0,\ 4\rangle$	$\langle 4, \infty)$
$f(x) = \frac{1}{4}x^4 - x^3 - 2x^2 + 3$				

(c) Compute the first derivative of the function.

(d) Determine the slope of the tangent line to the graph of the function intersecting in the following points; write it in the following table:

x_0	-4	$-0,5$	3	5
$f'(x_0)$				

(e) Use GeoGebra to create the graph of the function's first derivative.

(f) Specify intervals on which the first derivative is positive or negative. Write it in the following table:

interval				
$f'(x)$				

(g) Compute: $f'(-1),\ f'(0),\ f'(4)$.

Final task: Draw conclusions from your inquiry.

Figure 13.3. Worksheet 1 on monotonicity of a function, taken from the course Mathematical Analysis 1

Expected prior knowledge of students: elementary real functions of one variable and its properties, ability to compute a derivative of a given real function, tangent line and its slope (understanding the relationship between the slope of the tangent line and the function derivative at the given point); practical experience with GeoGebra or any other application for plotting graphs is helpful, but not necessary.

Goals: Performing activities of the unit enables students to

(1) recall the knowledge and skills they have received during the previous lecture and use it during performing activities described on both worksheets;
(2) make observations, formulate findings and justify them in a smaller group or during the whole class discussion to identify the relationship
   - between the first derivative and monotonicity including local extrema
   - and between the second derivative and convexity/concavity including points of inflection;
(3) get to know/remind the main features of GeoGebra.

Both worksheets are meant as activities to introduce students with methods to search for the intervals of monotonicity and points of local extrema, and for the intervals of convexity/concavity and points of inflection. But they should not stand as the only example how we can investigate these properties. That's why the final question is open and gives students or a teacher possibility to pose difficult questions. These worksheets need to be complemented with other, more complicated examples of functions, that make students think about points for which the function is not defined or the first/second derivative doesn't exist or the first/second derivative doesn't change the sign despite the fact it is equal to zero.[5]

[5] See the unit scenario: the session was supplemented by additional parts in which we focused on more complicated examples and explained both methods in more detail.

Worksheet – Convexity/concavity of a function

Let's consider the function $f(x) = \frac{x}{3-x^2}$ and its first derivative $f'(x) = \frac{3+x^2}{(3-x^2)^2}$.

(a) Make the graph of the function $f(x)$ using GeoGebra.
(b) Compute the second derivative of the function.
(c) Specify intervals on which the second derivative is positive or negative. Fill the following table

interval				
$f''(x)$				

(d) The graph of function f goes through the following points

$$A = [-1, -\tfrac{1}{2}],\ B = [0, 0],\ C = [1, \tfrac{1}{2}].$$

Determine equations of the tangent lines in these points and add their graphs on the GeoGebra canvas.

e) Look at the neighbourhood of the points A, B, C and compare the mutual position of the function graph and the tangent line in these points. Try to research how is this geometric relation connected with the sign of the function's second derivative.

Final task: Draw conclusions from your inquiry.

Figure 13.4. Worksheet 2 on convexity/concavity of a function, taken from the course Mathematical Analysis 1

13.5.3. IBME Unit Monitoring and Evaluation. We asked our colleagues David and Hanna to observe us during the sessions on monotonicity and convexity/concavity of a function. We sent them the worksheets in advance and described how we planned our session. We also specified the goals of the unit and how students should work with the worksheets. After the sessions had been realised we spent a few minutes discussing our first-hand impressions. Later on, we all met together and evaluated our sessions.

Luke's reflection (after the lesson observed by Hanna):

> Thanks to Hanna I have an idea about time spent on both worksheets and other information concerning the activity of 18 students who attended the seminar. The first worksheet took us 30 minutes in total. Some students worked individually, and some made groups. I was prepared to give them a paper sheet with a graph of the function in case of problems with GeoGebra. Few people asked me for that, but most of them worked with GeoGebra. The first student finished all tasks after 11 minutes, the last one after 22 minutes. We then spent several minutes discussing. I remember a lot of students were active during the discussion and came to the right conclusions without my help. I felt this IBME task fulfilled my expectations: the students were then able to work independently when solving examples on monotonicity and local extrema of given functions.
>
> The second half of the seminar was dedicated to the worksheet on convexity/concavity, again with the same setting as in the previous case. Most of the students used GeoGebra, one group asked me for the paper sheet with the graph of the function. We spent 40 minutes in total to solve all tasks and to discuss. I recognised a serious problem when students were trying to find the second derivative of the given function. Only 2 of 18 were successful; all others needed my help, so I had to compute it on the board in front of them. This activity took us a lot of time. There was one other computation in the end as the students were asked to find equations of tangent lines for three points. Some of them had problems again, but it was much better and not so time-consuming.
>
> After 30 minutes, I asked all students to summarise their inquiry. Maybe due to the complications with computations they seemed to be tired and weren't very active.

I may add another reason for their inactivity: for most of them the terms convexity/concavity and inflection points of a function were new unknown things, therefore it was not easy for them to understand the relationship between the second derivative and the position of the function's curve and the tangent line in the given point or interval. It was then difficult for me to convince them to interact and try to come with conclusions. After my own summary of the worksheet's results one student finally answered correctly and explained the relationship of the second derivative and convexity/concavity including points of inflection. Then we had about 15 minutes to solve other examples on convexity/concavity and inflection points of given functions.

Lenny's reflection (after the lesson observed by David):

I think both classes were successful. We did not finish everything but students became so engaged by inquiry that I did not want to interrupt them. In the beginning we finished examples on l'Hôpital's rule application. Then I gave them the worksheets on monotonicity and they were able to finish all tasks. After that I selected a function and asked them to find its intervals of monotonicity and local extrema. They worked independently without my help and succeeded.

At the end of the seminar, we started with the second worksheet on convexity and concavity. We did not manage to pass it all and stopped the work after task (c) to specify intervals on which the second derivative is positive or negative. I asked the students to finish the rest at home and we will summarise the activity during the next seminar.

Several students confirmed it was useful to write all three tables (the worksheet on monotonicity) on the board, one below each other. They were asked to fill the missing cells and then quickly find the right conclusions.

David told me after the seminar that it would be better to formulate the inquiry-based questions[6] more precisely so it can help students to come up with conclusions more quickly and easily. We both confirmed it is better to show these inquiry-based questions at the end of the worksheet's activity, so the students can better organise received ideas.

Next time I would choose a more simple example on convexity/concavity so students would not spend so much time on differentiating. They struggled to find the second derivative, on the other hand, they should be able to do so, but for some of them it had been the first experience 14 days ago.

13.5.4. IBME in Mathematical Analysis 1—General Conclusions. It was our first experience with inquiry-based teaching of mathematics. We created a small community of inquiry and worked together when designing, implementing, and evaluating IBME tasks. We appreciate the feedback from David and Hanna, who came with interesting ideas. As we wrote before, all IBME tasks served to repeat key terms from the previous lectures and met this goal from that point of view. Unfortunately, we were not able to add more IBME tasks during the second half of the semester due to lack of time.

The second main goal of seminars was to introduce students with procedures on computing a limit of a function, its derivative, finding local extrema, and so on. These are the skills students should know how to carry out and we tested them during both credit tests. As we struggled with time in the end of the semester, we focused on demonstrating students all procedures mentioned above and didn't have possibility to add more IBME tasks.

During the first phase of implementation we also recognised weak points of the IBME tasks we had designed:

[6]Lenny meant invitations to make conclusions on students' work formulated as the final tasks on both worksheets.

- In some cases (see our reflections in Sections 13.5.1 and 13.5.2) we formulated subtasks or questions too generally or we were not accurate enough, therefore students undertook the inquiry in a manner not directing towards valuable conjectures or gave so many irrelevant details not helping them to generalise their remarks.
- We did not estimate the time needed to work on both worksheets during the IBME unit well. Next time we may save some time if we do not ask for so many subtasks which are complicated or time-consuming but not relevant in a view of our educational goals that we described above.
- Students experienced difficulties when working with GeoGebra on smartphones. For some of them it was the first time they actively use this application and they had troubles entering functions into edit fields or it took a long time for their smartphone to respond and display a graph. We both reflected we should have recommended them in advance to take a tablet or laptop with them as it would have been probably much easier for them to work with GeoGebra on such a device.

Next time we can re-design all IBME tasks and the unit so that we can reach the goals we determined more effectively. Sometimes, less is more. And of course, it is better to use more simple examples so that students are not much occupied by particular activities that are not relevant to the final goal.

There is one added value with regard to the future employment of the students. As they will probably become teachers of mathematics in secondary schools, it was interesting for them to try IBME activities themselves and think about the pros and cons of this type of teaching/learning process.

13.6. Mathematics 2: The Experience of Marge

Within the PLATINUM project, several partial IBME activities were originally prepared for the Mathematics 2 course. However, due to the transition to online mode, most of them were omitted in 2020-2021. They required moderated group discussions, which is difficult to implement now (the university-wide introduced online learning platform does not support break-out rooms). So instead of these short activities, students completed a team semester project this study year. Eighteen groups consisting of three or four students chose one of the given problems (applications of linear programming in economics, finance, or management). The steps necessary to complete the task include building a mathematical model of the problem, choosing an appropriate software, using it to solve the problem, interpreting the results, and answering additional questions. An example of one of the assignments is shown in Figure 13.5.

The tasks were prepared by one of the course instructors in the first two months of the semester. Case studies from selected textbooks of operations research were used for the preparation of the project assignments, but the formulations were adjusted to change the procedural tasks into IBME activities. This means conversion of the sentence "Apply the sensitivity analysis to find the stability region for the capacity constraint" to "Try to guess what happens when the capacity of the resources is reduced by three units." The problems, as well as the way they were assigned to students were discussed with the rest of teachers at one of the meetings in the middle of the semester, so the expected outcomes and the plan for the activity implementation was agreed on:

The activity is designed to arouse interest in the topic and introduce new concepts connected to the area of linear programming. The applied character of the tasks is supposed to help the students build connections between theory and practice. When

Task 1 Production mix of an engineering company

An engineering company is using two processes (grinding and drilling) for the production of five kinds of products. The individual products generate following unit profits (CZK):

PROD 1	PROD 2	PROD 3	PROD 4	PROD 5
550	600	350	400	200

Each of the products has a different processing time using the individual production processes. The times are listed below (in hours). The dash indicates when the process is not needed.

	PROD 1	PROD 2	PROD 3	PROD 4	PROD 5
Grinding	12	20	–	25	15
Drilling	10	8	16	–	–

In addition, the completion of each of the products requires 20 hours of assembly line staff time. The factory has three grinding machines and two drilling machines, and the machine operator has a six-day working week with two shifts of 8 hours each day. Eight workers are employed in the assembly, each working one shift a day. Your task is to find out how many products are to be produced in order to maximize the overall profit. Formulate the model, find the optimal solution and find out which resources will be completely depleted and which will leave unused capacity. Next, try to answer the following questions:

1. *How much higher should the price of product 3 have to be to make it worth producing?*
2. *What is the value of the additional hour of grinding?*
3. *What is the value of the additional hour of drilling?*

Figure 13.5. Assignment of a team project, course Mathematics 2

deciding on the implementation of the activity, we tend to be open in almost all considered dimensions of inquiry learning: we let the students involve themselves freely concerning exploration, making observations, planning investigations, justifying, and posing questions; the only exception is the closed form of formulations of findings. The cooperation and using tools are essential for the completion of tasks.

Students elaborate on the solution within their groups. Intended working time for the activity is 10 to 15 hours in total, but it can be split between more people. The group work is realised outside the lessons during November; students can consult the teacher every week during special office hours to discuss their progress. In the first two weeks of December, presentations of student solutions take place during the lectures (nine presentations of ten minutes length each week). It is realised as a videochat, one student is selected in each group to speak for the team and share the computer screen with a PowerPoint presentation. After the presentations, the teacher generalises the findings obtained from the problem solutions and makes students familiar with concepts of the duality in linear programming, sensitivity analysis, shadow prices, and other topics that could help answer the questions asked in the tasks.

13.6.1. Evaluation of the Activity. As the teacher was present only at some parts of the whole process (consultations and final presentations), the observation was replaced by student self-assessment in this course. The presentations were not recorded because some presenters feared that the recording would make them nervous. After the presentations, we have sent the students a feedback questionnaire to evaluate the activity. First, students expressed their overall opinion on the activity using a quantitative scale, see Figure 13.6.

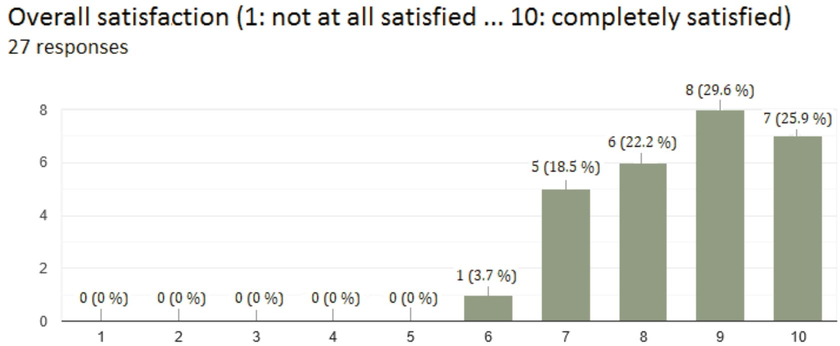

FIGURE 13.6. Students' evaluation of team projects in Mathematics 2.

Students also answered open questions, and they reported on their progress in solving the problems. They should state in particular:

- how long it took them to solve the task;
- what the level of cooperation was in their group
- what difficulties they encountered in carrying out their tasks,
- the means by which they overcame these difficulties;
- what knowledge and skills they think that they acquired during the activity; and
- what they consider to be the greatest benefit of this way of teaching.

Below, we provide the translations of feedback from four teams selected to represent the variety of answers of the whole student cohort (18 teams):

Group 1: I think we spent too much time preparing the presentation. I would consider it more beneficial to practice on easier optimisation problems (more exercises covering typical problems that may appear in the final test—solving problems using graphical method, etc.).

Group 2: The cooperation inside our team on the assigned task was great. It took place via social networks, thanks to which we contacted and discussed on a regular basis. We used written conversations and up to it made about three calls. During the first call we tried to figure out how to model the problem: what variables to choose, and how to insert them in the objective function and the constraints. This was the hardest part for us. That's why we agreed to have two days to think it over, and then to meet again and agree on further action. We tried to look for similar examples and possible solutions on the internet and consulted with our friends and the teacher. That helped us a lot to complete the task.

We are not sure if the project task would help us in further study or in writing final theses, although it is undoubtedly good for better understanding of the theory covered by the course, maybe also for improving our critical thinking.

Group 3: We tried to come up with a solution in the first video call, but we didn't figure it out. The first video meeting resulted in various suggestions on how we will be able to work on the solution, as well as tasks for each member of the group (to watch the lecture, get acquainted with the Solver add-in in Excel and conduct a search on the issue). During the individual work, we shared suggestions or relevant links to articles and videos so that all members have access to them, and then we analysed and discussed them. After the second video session, based on the knowledge gained through self-study and discussions, we managed to model the problem. We implemented the solution with the help of the Excel add-in Solver.

When preparing the presentation, we came across an additional issue, which was to write an objective function and constraints of the problem using a double sum notation. Again, we followed a similar procedure as in solving the problem itself. We first shared the information we found relevant, links to articles, explanations from the textbook and other research results. Subsequently, we had another video call using Microsoft Teams and together we worked it out. The total length of video calls was approximately 3.5 hours. Together with other activities, we spent about 10 hours working on the project. We consulted the teacher to check the solution. The problem seemed complicated at first, but after getting acquainted with the Solver add-in by Excel, we changed our minds. Writing a double sum seemed to be the biggest problem for us. However, we also consider this as the biggest benefit, because this part of the task combined the theoretical knowledge that we gain in class with an application on a real particular case. Another benefit is that we had the opportunity to get to know another useful Excel add-in we had no previous experience with.

Group 4: Our project task showed us how we could make decisions in the future if we want to invest our money in various financial instruments and at the same time, maximise profits. We realise that this type of problems does not capture the real world with 100 percent accuracy, but we think that such types of problems are an excellent tool for learning mathematics through real-life problems and thus arousing greater interest in mathematics. We were thinking about how we could make this project task closer to reality, and we thought of taking into account the risk factor and solving this task for more scenarios.

As can be seen, the opinions varied significantly. Some students did enjoy the task but did not find it very useful (Group 1 and two other groups). They found it too difficult and not connected to the final exam. On the other hand, some students suggested the inclusion of additional constraints in the problem leading to a more complex problem structure with higher application potential (Group 4). Six more groups mentioned that motivation by the real application made them more engaged in the activity. The majority of students considered the activity beneficial and declared that it helped them to increase the level of their understanding of the topic in general (see Groups 2 and 3). Students of Group 3 and five more groups found another benefit of the activity in getting acquainted with the recommended software tool for solving optimisation problems. Although it was not planned as a goal of the IBME activity, it helped to arouse more interest. There was almost total agreement on the usefulness of working in teams. In general, students appreciated the collaboration; only in one case a team reported a non-responding team member, but after the intervention of the teacher this student started to participate in the team work. It was difficult to check whether the tasks took the time intended for their elaboration. With some exceptions (Group 3), the teams didn't report on their work progress properly, only some of them vaguely reported that the problem was too time-consuming (Group 1 and three more groups).

At the end of the semester, the lecturer together with the course instructors discussed the results and students' feedback and the conclusion was to keep the activity in the slightly modified form for the next year. If it is possible to transfer it from distance to in-person teaching, we will be able to better evaluate the activity. The only major modification considered for the next course is the inclusion of a simplified version of the project problems in the final exam to involve also the students motivated only by the successful completion of the course. However, we are aware of the fact that the activity is time consuming and its implementation in the course with an extensive curriculum can only be incorporated because most of the process takes place outside the class.

13.6.2. Concluding remarks by Marge. I have started introducing IBME elements into my courses Mathematics and Mathematics 2 more systematically last year.

The activities have been accepted rather positively by students, and my enthusiasm for other efforts grew. This enthusiasm, however, slows down during this semester, because I feel very limited in finding a way how to squeeze IBME activities into a very condensed course, taught on an online platform. In any case, I welcome the involvement in the PLATINUM project because it allows me to prepare my lessons more systematically, to reflect on my teaching, and to exchange experiences with other colleagues. However, I do not expect that the project PLATINUM will help me change my own lessons radically. I guess that one of the main barriers for more intense using of inquiry in teaching is the students' mindset and their expectations. In the opinion poll, students repeatedly report that there was "too much theory and not enough exercising." Even if they enjoy the activities, they usually perceive the time devoted to building conceptional understanding as "lost" or "inefficient."

I am aware of the usefulness of IBME approaches in teaching mathematics but I consider the state of mathematics education at all levels of schools in the Czech Republic to be deeply underdeveloped in this respect. Elementary schools focus on preparing students for typical problems in secondary school entrance mathematics tests and a similar situation is at the next level of education. Thus, if the success rate of elementary schools is measured by the percentage of pupils in high school admissions and the success of secondary schools by the percentage admitted to higher education, it is no surprise that schools focus on training students for standardised problems. And this is naturally the same thing what students expect from their mathematics teachers when they enter university: to train them for the success in standardised exam tests. So, driven by these expectations, students are not able to fully appreciate the benefits of pedagogy aiming at deeper conceptual understanding. In my opinion, a good design of assessment and exams testing rather conceptual than procedural knowledge could break this status quo. It is of great importance to improve the situation also at the lower levels of education.

13.7. Summary

The functioning of the community of inquiry at Masaryk University is special as the individual team members work in several workplaces and participate in the teaching of many different subjects which makes the coordination of our activities more difficult. Meetings and IBME discussions are organised on different levels:

- observations in the lessons;
- small teams of people teaching the same course: rather informal discussions on a regular basis (approx. every two weeks). We don't take minutes, but we share material in the university information system;
- irregular meetings of the whole MU team, usually together with BUT team (structured, recorded); and
- events for a broader community, including people outside MU.

Within our community, IBME elements were introduced into large courses on mathematics or statistics for non-mathematicians (mainly future economists or teachers at primary schools). We described three different cases of IBME implementation. It turned out to be quite challenging as our courses have comprehensive curricula and a tight time plan. Another problem turned out to be the spatial fragmentation of individual workplaces of the university, when, for example, due to the need to move between buildings, it was not possible to immediately provide feedback from the observer to the teacher after the monitored hours. An equally important problem that we have not yet been able to solve, was the resistance of some teachers to the use of

IBME principles in teaching. As our experience from the course Mathematical Analysis 1 has shown, the inclusion of IBME only in a part of teaching (seminars) without alignment with the overall objectives of the subject, the method of evaluation, etc., is limiting the benefits of this approach. Nevertheless, even the small progress matters. Quoting Jaworski (2008, p. 313),

> in an inquiry community, we are not satisfied with the normal (desirable) state, but we approach our practice with a questioning attitude, not to change everything overnight, but to start to explore what else is possible; to wonder, to ask questions, and to seek to understand by collaborating with others in the attempt to provide answers to them (Wells, 1999).

To conclude, let's go back to the issue of insufficient professional development at our institution. Our group's specific feature is that our educational background is either mathematics or statistics; we have no mathematics educators among us. The majority of us have not even been introduced to the basics of the theory of pedagogy. Participation in the project helped us a lot to approach the educational process more systematically, to become aware of our teaching methods, to share the experience and collaborate as members of an emerging community of inquiry. So we may say that it adds to filling the gaps in this area. Our experience with IBME units can be valuable for other colleagues at our university if we continue disseminating IBME ideas.

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