Inquiry in University Mathematics Teaching and Learning: The PLATINUM Project
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CHAPTER 10

Introduction to the Case Studies in PLATINUM

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PLATINUM is a project in which the main focus is the development of teaching and learning mathematics through an inquiry-based approach in which our students, through their own inquiry in mathematics, engage more deeply and develop more conceptual mathematical understandings. The project spans seven countries with partners in eight universities. In each of these universities the partner team has engaged with ideas about inquiry-based learning and teaching in mathematics and, in so doing, members have developed their own practices.

In our submission to the EU Erasmus+ programme, we promised a book in which each partner would write a case study, from their inquiry community, in which they would discuss their learning and development through engagement with the six Intellectual Outputs of the project (see Chapters 2 and 5 and below).

This chapter provides an introduction to these case studies and a brief perspective on the main elements of each case study.

10.1. Inquiry-Based Mathematics Education—Basis for Our Case Studies

10.1.1. The Idea and Nature of a Case Study. We anticipated (and indeed expected) that each partner team would take a developmental focus related to their own history and academic culture and that these would therefore differ from one country, from one institution to another. As the project has developed, it has been interesting to see these focuses emerge in relation to our agreed three-layer model (Chapter 2) for exploring inquiry-based activity. The idea of a case study was intended to give each partner an incentive to reflect on their development in a very local and personal way that could provide readers with an insight into their developing experiences. With communication through meetings, workshops, dialogue and writing, a vision of these local experiences began to emerge and we started to perceive a local 'essence' in that experience. We were all encouraged to write 'narratives' that captured our own experience of participating in this project, and it was interesting to see how the narratives developed with the project. Early narratives were written in a rather formal manner in which the narrator could be seen as an outsider reporting on an observed event. Gradually we started to see individuals taking an insider role, reporting their own activity, decisions, issues and feelings. It is these more personal narratives which provide a deeper insight into the essence of participants' experience. In writing our case studies, we were encouraged to include extracts from these more personal narratives and you will see how this is done in different ways as you read.

10.1.2. Our Developmental Processes. When we began this project, every partner group and every individual within these groups had a vision of the project and a corresponding vision of inquiry-based mathematics education. Undoubtedly, these visions differed. Some were related to extensive experience of developmental activity in inquiry-based mathematics learning and teaching (often abbreviated to IBME—Inquiry-Based Mathematics Education). This may have included theoretical perspectives and a knowledge of related literature. Others were coming new to IBME, perhaps with some feelings of uncertainty, even insecurity. Overall, it is fair to say that we were all concerned about our students' learning of mathematics and the extent to which they engage with mathematical concepts. The levels of experience varied considerably, together with perspectives on what IBME might look like in practice, what the difficulties might be in achieving it, and whether it was really possible to realise our aims in the local environment and culture. Project organisation has, fundamentally, respected these differences and tried hard to work with them sensitively. While the Intellectual Outputs (IOs), as written in our proposal, have provided guidance for our developmental activity in each partner group, there has been space and encouragement to work locally according to our own visions of what could be involved. For example, IO1 provided a theoretical framework, IO2 an expectation to work together and form an inquiry community in our partner group; IO3 gave a lead on the design of inquiry-based mathematical tasks for use with our students. The extent to which we have focused on each of the IOs has been for each group to decide. Sharing our activity and perspectives through our project workshops has enabled us to grow in understanding, both as a project community and as partner communities, of what might be possible and how we would interpret the expectations of the IOs. The local teams, each developing as an inquiry community, have explored possibilities in their own ways and the result, we believe, offers a richness of experience and outcomes. The diversity of essence makes clear for readers the many ways in which IBME can be interpreted and experienced at university level.

10.1.3. Issues and tensions. For all of us in the project, the developmental process has had many elements, paths, directions and experiences: many satisfying, rewarding, illuminating; others more challenging, disturbing, worrying. When we try out new practices – new ways of presenting mathematics, new activities for our students, new ways of being a teacher, new ways of expecting students to learn—the outcomes may not be what we had envisaged or hoped for. While this is likely to be a great learning experience, it can also be depressing and demotivating. Issues can arise due to factors such as the available lecture theatre or tutorial room, students' responses to what we have asked them to do, technical limitations, educational infrastructure. They can also arise due to our own ways of presenting ourselves and interacting with students. In our awareness of these possibilities, we may unwittingly influence our students against the practices we would like to promote.

What is perhaps important is not our narrow judgmental evaluation of such outcomes, but our *inquiry* into the factors involved and how any of these might be changed to afford outcomes more in line with our goals and associated visions. This can be where *inquiry*, as well as being a factor in and for mathematics and its learning, can be at the centre of our developmental process. If, when we plan a lecture or seminar, we see our actions as a design stage for something we will try out and reflect on its outcomes, we may become aware of a range of factors that were not visible before, but which can be modified subsequently. The outcomes inform us, give us insight into what is possible or not, and why. In these respects, we work in the second layer of our model. This approach is sometimes called 'action research' (Elliott, 1991) or 'design research' (Design-Based Research Collective, 2003) depending on how it is carried out. When such an approach is informal, the developmental outcomes are informative and encourage us to reformulate and try again. When the approach is more formalised, it becomes a research approach, fulfilling the criteria for validation, verification and trustworthiness of interpretation and formalisation. In these respects, we are working in the third or outer layer of our model and producing outcomes that can be shared more widely to inform our professional and/or research community (see Chapter 2).

In these case studies we find examples of all the elements mentioned above. We hope that they inform and inspire you as reader to engage with IBME in your own environments, inquiring into your own practices and their development and making possible for your students to gain a more inquiry-based perspective of mathematical concepts.

10.2. Elements of Our Inquiry Activity in the Case Study Chapters Which Follow

10.2.1. References to Didactics and Pedagogies. Fundamental to all teaching, even if not stated or recognised overtly, are concepts and practices under the headings *didactics* and *pedagogy.* These terms are perhaps redolent of education courses in teacher education programmes. However, mathematics-teacher-education programmes exist primarily in pre-tertiary education and, to date, there are far fewer educational programmes directed towards teachers in higher education. As Winsløw et al. (2021) have pointed out, there is a growth of general educational programmes in universities, although much less that is subject-based (e.g., mathematics-based). General educational programmes tend to deal more with pedagogy than with didactics which is highly subject related.

Just briefly, didactics of mathematics deals, practically, with the ways in which teachers who *know* mathematics transform this *knowledge* into activities for learners. Such activities include listening to exposition or explanation from a teacher, making sense of examples provided by the teacher, working on mathematical problems (perhaps with their peers), engaging with mathematical tasks carefully designed by the teacher to focus attention on key elements of mathematics. Theoretically, didactics addresses the relationships between the engagement in mathematical activity and the learning of mathematics and is the province of 'didacticians' of mathematics in university education. In PLATINUM, we have focused rather more on the practical side of didactics than the theoretical side.

In contrast, pedagogy in mathematics learning and teaching focuses on the ways in which activity with students is organised. So, for example lectures to several hundred students are a form of pedagogy. Organising students into small groups to work on carefully designed tasks or problems is another. As with didactics, pedagogy has its own theoretical bases, often addressing learning and what it means to learn. For example, general theories include constructivism or behaviourism; more particularly related to mathematics are the theories around problem solving, or in our case inquiry-based learning of mathematics (IBME). As with didactics, pedagogy in PLATINUM has been much more practically focused.

In the chapters which follow, you will find considerable focus on didactics and pedagogy, even where these terms are not used explicitly. This is because we are fundamentally addressing what teaching and learning mathematics mean for us. Especially a pleasure in reading the case study chapters has been the ways in which different authors have explored the educational literature to inform their writing, or have used Information and Communications Technology (ICT: see below) to find new ways of exploring mathematical concepts. It is clear from reading these chapters what a valuable experience PLATINUM has proved to be in terms of our own learning in these areas.

10.2.2. The Three-Layer Model—a Basis for Each of the Cases. The PLATINUM project can be seen to draw on a range of theoretical perspectives informing inquiry-based learning and teaching and relating to the educational perspectives of those speaking or writing. Despite this theoretical diversity, at the centre of PLATINUM has been the *three-layer model* which has provided a basis for both theory and practice in our activity. In our proposal to Erasmus+, we promised to develop a (theoretical) framework which could guide our work in PLATINUM: the three-layer model provides this framework (Chapter 2).

Much of the published work about inquiry-based teaching and learning relates to pupils or students in classrooms and their inquiry into mathematical concepts, as well as the design of tasks for this purpose. This has of course also been central to PLATINUM. However, PLATINUM has gone further to see teachers' design of tasks as an inquiry process in which we develop our knowledge of task design through an iterative, cyclic, process (plan, act, reflect, feedback) in which we refine our plans at each cycle. The result might be a prototype task developing its potency at each stage or the experience gained by the designer in seeing the task in use, or indeed as a teacher putting it into use (see Chapter 12). In all these cases inquiry in each cycle leads to new knowledge and awareness for the teacher/designer. This is a (natural) professional development process which does not depend on formal training. The third layer of the model makes this whole process less intuitive and more explicit. We seek to justify each stage of the process, presenting evidence for our claims for learning and development, perhaps through the reflections of those involved or by analysing data from the inquiry activity.

10.2.3. Community of Inquiry (CoI). *Community of Inquiry* is a fundamental concept in our theory of inquiry in PLATINUM. It spans the entire three layers of the framework and crops up in all of the chapters below, in some cases very frequently. We have taken the idea of CoI from the literature (e.g., Cochran Smith & Lytle, 1999; Jaworski, 1998; Wells, 1999) and have built its use in PLATINUM on projects in Norway in which teachers at school level worked collaboratively with didacticians in a university to develop the mathematics learning of pupils (e.g., Goodchild, 2008; Jaworski, 2008). PLATINUM is, we believe, the first use of this theory in the teaching and learning of mathematics in university education. The many references to CoI in these chapters provide evidence that this theoretical construct (CoI) has been taken up in practice by these university teachers and assimilated into their thinking about and language of teaching development. A CoI can consist of 2 people or 20 or 200 people; its characteristics are that its members inquire into their learning and into their practice. So, we can have a CoI between students learning mathematics, between teachers designing a teaching unit, or between didacticians and teachers, together, analysing the learning outcomes of teaching. Possibly the best way to find out what CoI means is to read these chapters and build up a picture from their differing ways of describing this concept.

10.2.4. Working With Students Who Have Identified Needs. As teachers we are all aware of the great diversity of needs of our students, although not always confident that we know how best to recognise needs and provide support. In Chapter 4 we read:

> Taking this diversity into account, we prefer to use the social model of disability whereby difficulties are seen as a product of social circumstance, removing the onus from the individual and giving the responsibility for inclusive learning environments to educational

> institutions. "This is in contrast to the medical model of disability that concentrates on the impairment as the cause of the disability" (Drew, 2016, p. 30). (p. 50)

In PLATINUM, led by colleagues with expertise, we have been introduced to a variety of particular needs (e.g., physical needs caused by sight or hearing loss, neurodiversity including dyslexia, ADHD) with advice and discussion about how the social setting (e.g., of mathematics teaching) can be prepared or adjusted so that we do our utmost to *include* the students and *address* their needs. Chapter 4 offers a comprehensive perspective on what *addressing* might involve. *Inclusion* is more complex. As Chapter 4 asserts:

> The social model requires that educational institutions take on responsibility and break down barriers in order to ensure an inclusive learning environment. (p. 50)

For many colleagues in PLATINUM, the contents of Chapter 4 are new, informative and demanding on teachers, especially where the institutional position is not so clear.

Our focus has been twofold: (a) in the design of tasks and teaching units, ways in which the design takes into account a diversity of needs (see Chapters 6 and 12); and (b) regarding pedagogy, it has been up to each of us to use pedagogies that are as widely inclusive as possible. Examples include, the design of teaching such as that described in narratives in Chapters 11 and 13; using computer software to provide alternative insights in Chapter 15; dialogue between teachers and students in Chapter 18. While we do not claim to have developed extraordinary expertise in considering diverse needs, our awareness of particular needs has been enhanced and we have begun to consider inclusion in our inquiry more generally, particularly in the second layer of our model where we inquire into new teaching approaches alongside new design of tasks etc. As we read in Chapter 4:

> While students undertake inquiry-based instruction, teachers inquire how to implement some of the Universal Design ideas into their lectures and seminars. Such development is continuous and clearly needs the feedback not only from students but also from experts on inclusive education in order to evaluate the effectiveness of implemented recommendations and plan other modifications of the course. (p. 68)

We hope our expertise will grow through our inquiry and invite readers to use inquiry as a means of including students in mathematics more conceptually.

10.2.5. Use of Information and Communications Technology (ICT). In our proposal to Erasmus+, we indicated that one of our areas for inquiry would be the use of ICT in developing inquiry-based tasks for teaching and learning. We are aware that this is a major area of research in school-based development, but not yet so in university education (although, see Gueudet, 2017). In PLATINUM we have incorporated digital tools and methods into our design of tasks or our teaching approaches. For example, in Chapter 12, we see the use of a free software for generating graphs or geometrical situations for exploration, GeoGebra,[1] and the use of an open source programming environment for exploration of dynamical systems, RStudio,[2] along with a cloud-based environment for students' on-line inquiry-based learning, SOWISO.[3] In Chapter 15, we see a package called Autograph[4] used with students to inquire into operations with complex numbers, providing a pictorial way of 'seeing' the complex concepts that are involved.

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[1] www.geogebra.org

[2] www.rstudio.com

[3] www.sowiso.com

[4] https://completemaths.com/autograph

During the COVID-19 pandemic, teachers were required to teach on-line according to government regulations interpreted by our universities. The use of video to present lectures or demonstrations became more common as did the use of communication platforms like Zoom[5] and Microsoft Teams[6] to organise online meetings, to allow online collaboration of students, and to provide group chat functionality and screen sharing (e.g. an online whiteboard or a OneNote document). Flipping the classroom was a strategy some partners used for introducing inquiry teaching in an online setting.

An important consequence of these varied uses of ICT, many of which were intuitive or inquiry-based, is that we have become more aware of and more experienced in alternative approaches to stimulating our students' learning of mathematics. The common lecture is no longer the only way of teaching at our disposal. We see inquiry opportunities in all these digital approaches.

In consideration of the use of ICT in *school* mathematics, there has grown a wide literature on ICT potential and use, much of it theoretically based. Here we find theoretical concepts that could well apply to teaching mathematics at university level. PLATINUM has not so far engaged with such theory, but inquiry-based mathematics education at university level could valuably do so.

10.2.6. Inquiry-Based Tasks in All the Chapters. As suggested above, in these case study chapters you will find many examples of inquiry-based tasks of differing sorts, serving different purposes. Some are first attempts to think about a different style of question from the more traditional questions we are used to. Some are more sophisticated in their design, being created to fulfil different didactical purposes. The teaching units associated with these indicate something about the particular task, the pedagogy associated with the task and the overall didactic expectations related to students' learning. Chapter 6 has provided a comprehensive discussion of such tasks and teaching units, their design and use. In addition, each partner group has provided more detailed examples of tasks and teaching units which can be found on the PLATINUM website.

One way to begin to think about teaching through inquiry is to start with tasks which have inquiry-based characteristics, try them out with students and learn from students' responses. This can lead to the adapting of new elements of didactics and pedagogy more generally in teaching. An issue here of course is that students themselves are not used to such tasks and teaching and can be very resistant to it. They may see it as making unfair demands on them – requiring new forms of involvement without telling them precisely what is required, what is right and what is wrong. Many of the teachers in PLATINUM have faced such responses from students (see for example, Chapter 11). Finding ways through this didactic/pedagogic minefield can be seriously discouraging, making us lose confidence in what we want to achieve. One way to cope with this and come through it is to discuss it with colleagues in a CoI, share teaching approaches and the insights we learn from using them, and find ways of convincing students that the whole inquiry process is worthwhile. It is worth recognising that the PLATINUM insights and findings have developed over three years of working in CoIs and exploring the use of inquiry-based tasks with our students—this has not happened 'overnight'. Several of the cases below show evidence of such development.

10.2.7. Reference to the IOs. As explained in earlier chapters, the ERASMUS+ programme required us to declare Intellectual Outputs which would be developed by PLATINUM; we declared and developed six IOs as described in Chapters 2 and 5.

[5]`www.zoom.us`

[6]`www.microsoft.com/en-CA/microsoft-teams/group-chat-software`

The responsibility for leading activity within the IOs was spread across the partners. Each IO has at least one chapter dedicated to it in Parts 1 and 2 above. However, there are many overlaps.

IO1 and IO2 permeate all the cases, bringing a rich panorama of practice to the more theoretical elements of inquiry, the Three-Layer Model and Community of Inquiry. The tasks-based substance of IO3 also permeates widely, with the cases showing differing scenarios and types of task. These three IOs might therefore be seen to capture the inquiry basis of PLATINUM. However, the other three IOs are no less important. IO4, focusing on professional development for new lecturers, sets the scene for supporting colleagues in developing inquiry approaches to teaching and learning. New lecturers are not the only ones needing support with inquiry-based teaching. As suggested above, even many experienced lecturers need some kind of support when seeking to use inquiry for the first time. The approaches suggested in Chapter 7 can apply to all lecturers: those more experienced with inquiry-based teaching can engage with their colleagues in the ways suggested in Chapter 7 and it is likely that all will learn from this activity.

The other two IOs are somewhat different. IO5, focusing on *modelling*, extends the focus of IO3 to the design of tasks of a modelling nature. This focus is especially valuable when working with students from other disciplines such as engineering, science or economics. Drawing teachers from these disciplines into the dialogue can be valuable in developing a more comprehensive approach to teaching these students rather than isolating mathematics from them, and hence encouraging students to underestimate its value to them.

IO6, focusing on evaluation, offers a perspective on gathering evidence from our teaching activity regarding the extent to which students are indeed learning what we have set out to teach. Presenting an example of an evaluation instrument, the IO addresses the approaches that partners have used to evaluate teaching and learning (see Chapter 9 and Chapter 18). As you read the case studies below you will gain some insight into how PLATINUM has addressed such issues. However, in this area there is much more to be done and you might consider how this relates to your own teaching and its outcomes.

10.3. Introduction to Each of the Case Studies

In this section, there will be a short introduction to the main aspects of each case study, drawing out the diversity and commonality of the experiences/essences discussed there. The purpose of these short abstracts is help you to decide where to start and which cases might be most interesting to your own thoughts on inquiry-based teaching.

Where we refer, below, to the *teaching* discussed, we use the term "teacher" to describe the person working with the students, avoiding terms like professor, assistant professor, lecturer or assistant lecturer unless the term is particularly meaningful to what is described. In most cases, we have found that the experiences reflected cut across these boundaries.

10.3.1. Teaching Students to Think Mathematically Through Inquiry: The Norwegian experience. In this chapter, we find an account of two teachers' very different experiences of teaching in which they introduce aspects of inquiry. The two teachers, both mathematicians, together formed a *Community of Inquiry*, with support from colleagues with more didactical experience. An aim for their teaching was to motivate students to take more responsibility for their own learning.

With reference firstly to a course on *Ordinary Differential Equations* (ODEs) for *engineering students* and secondly to a *Multivariable Calculus* course for *first-year bachelor's students*, we find examples of the mathematical tasks used and references to ways in which students have responded. Recorded dialogue from small-group activity with students provided insights to students' responses to their inquiry activity. The teachers each reflect on the influence the design of their course had on students, the challenges from institutional factors affecting outcomes and their own associated learning. We see here, mathematicians finding it useful to address the mathematics education literature within the PLATINUM frame of Inquiry Community; they demonstrate important elements for students' learning through inquiry and the challenges faced when students and inquiry do not seem to mix well.

10.3.2. Design and Implementation of an Inquiry-Based Mathematics Module for First-Year Students in Biomedical Sciences. In discussion of a first-year mathematics module for biomedical students, the three authors 'set the scene' of their *Community of Inquiry* (CoI), the roles of its members, and their aim to use inquiry-based teaching with students. They show how the structure and content of the module evolved through meetings of the CoI, each recorded for later analysis, and their use of digital inquiry (involving SOWISO, GeoGebra and R/RStudio). The inquiry-based nature can be seen through examples of designed tasks (in Calculus/ODEs) and emphasis on students' critical thinking going beyond traditional courses. The research literature in IBME and feedback from students raise issues in the CoI for teachers' reflection on their teaching, on the progress of the module, their use of ICT and their students' experience of a new style of teaching and learning. The redesign of the module for a second year demonstrated their opportunity for putting this learning into practice in changed circumstances.

10.3.3. The First Experience with IBME at Masaryk University, Brno. Mathematicians and teachers work together across disciplinary boundaries in a large and diverse team to introduce inquiry-based activity in courses in statistics and mathematics. Having no previous education for teaching, and no knowledge of inquiry-based activity, they bring readers into the collaborations formed within their Community of Inquiry and the ways in which practices developed. Reflections of members of the CoI on the elements of their inquiry into teaching, the tasks they designed and used and responses of students provide a rich tapestry of learning for both students and teachers. We gain insight into teaching of statistics and mathematical analysis in which traditional practices are modified or replaced in inquiry ways, informed by the research literature. Observation of teaching by others in the CoI encouraged sharing of experiences and issues, and promoted learning for all. The reality of juxtaposing traditional and inquiry-based practices raises many issues for the team and for their future development of mathematics teaching in the national and institutional contexts.

10.3.4. In Critical Alignment With IBME. This chapter focuses on a group of mathematics education researchers teaching a mathematics education course for student teachers in Germany. The researchers focus on the teaching and the challenges they face in addressing tensions between the theories they teach in the didactics of mathematics and the traditional practices, beliefs and customs in the educational system and the students they teach. The tensions are reflected at two levels: First, as symptoms of inescapable institutional-societal or current teaching-learning conditions, and second, as the contents of theories they teach, which constitute both obstacles and conceptual opportunities to learning. Starting from a mathematical problem involving questions about the nature of graphs with or without inflection points, they reveal a range of issues, both mathematical and sociocultural, that affect their relationship with their students and affect their own development as teachers and researchers. They weave the PLATINUM concepts of Community of Inquiry and Critical Alignment with theories from German educators and social scientists to present counterpoints between theory and practice in teacher education.

10.3.5. Two Decades of Inquiry-Based Developmental Activity in University Mathematics. Focusing on inquiry-based learning and teaching in projects in which they have engaged, a group of mathematics education researchers presents and reflects on examples from their own practices in research and teaching. Seeing themselves as a Community of Inquiry, they consider their inquiry-based activity with other colleagues and with students, reflecting on experiences from which they have learned as practitioners. These include the inquiry of a mathematician into making examination questions more inquiry-based; of a teacher-researcher designing mathematical tasks for her students involving computer-based inquiry; and of a teacher reflecting on the issues raised when working with groups of engineering students in traditional or inquiry ways. These examples recognise that inquiry-based teaching is not a simple matter, but its challenges create insights for learning about teaching.

10.3.6. Teaching Inquiry-Oriented Mathematics: Establishing Support for Novice Lecturers. Inquiry activity here draws on a long experience in university education of innovative teaching/learning activity in mathematics and mathematics education, involving mathematical problem-solving and forms of IBME. The chapter focuses particularly on professional development for new lecturers designed by a team of four experienced teachers with fields of research in mathematics or in mathematics education. The design process for the materials to be implemented is divided into four phases: Discover, Define, Design, and Develop. The chapter shows how these phases are applied, the activities in each phase, the mathematical tasks implemented by the new lecturers with students, and reflections of both lecturer and students on their activity and learning. An example of matrix factorisation illustrates the implementation of this process in some detail, showing its outcomes for lecturers and students in terms of critical attitudes to mathematics, to learning mathematics and to mathematical meaning and processes. Finally, the authors recognise how the whole process of design and implementation contributed to their own learning as teachers and researchers.

10.3.7. Development of a Community of Inquiry Based on Reflective Teaching. In one university, as this chapter reveals, there was no immediate group of people available for forming a Community of Inquiry (CoI) and alternative ways had to be explored, including inviting colleagues from neighbouring universities. A key expectation of the CoI was to provide observation of teaching to allow feedback and discussion of the observed teaching and learning. Colleagues observed each other's teaching and shared their practice. It allowed inquiry into diverse classroom settings, their affordances and constraints for teachers and students to be considered. Mathematical tasks were designed and shared. The Covid pandemic, with on-line learning and teaching, provided new environments to be explored. Overall, a relatively stable CoI was achieved that allowed reflective teaching to be shared and inquiry-based activity to be established in this community. Challenges, achievements, and experiences of the CoI are discussed, with narrative reflections from its various members.

10.3.8. Experience in implementing IBME at the Borys Grinchenko Kyiv University. In this chapter, we learn that an educational community was formed at the university, including colleagues from several universities, to address low motivation of students in choosing mathematics programmes and to share understandings of IBME. This Community of Inquiry (CoI) addressed issues related to conceptual versus procedural learning approaches drawing on a range of literature. PLATINUM members led the CoI in suggesting inquiry-based approaches to teaching and learning and one member led a course in Mathematical Analysis to enable the community to observe and address processes and issues. An open questioning approach was taken by this teacher with encouragement for students to address the questions and to ask their own questions. Extracts from the teaching and examples of students' responses suggested that teaching had motivated students and engaged their interest, thus also motivating the CoI to engage further with IBME approaches.

10.4. Concluding Thoughts

The PLATINUM partners invite you to read our chapters described briefly above. We believe that each one offers insights into engaging with inquiry-based teaching and learning. Although this book can offer only brief examples of inquiry-based tasks, inquiry-based ways of working with students, or inquiry-based theoretical analysis, you will find further examples on the PLATINUM website `https://platinum.uia.no`. It may be that you have experiences that could add to the richness provided in this book and on the website. And of course you can contact corresponding authors of chapters directly.

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