Inquiry in University Mathematics Teaching and Learning: The PLATINUM Project
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CHAPTER 9

Evaluation of Inquiry-Based Mathematics Education

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9.1. Introduction

The PLATINUM project is a joint effort to develop an approach for teaching and learning mathematics at university level that will improve the balance between procedural and conceptual learning of mathematics and build a community of inquiry that will disseminate this approach across European universities. It promotes inquiry-based practices while encouraging collaboration across regional, European and international institutions. This chapter focuses on one of the main intellectual outputs on the project: "Guidelines and recommendations for quality assessment in Inquiry Based Learning environment" (IO6). One of the main goals of the project is not only to provide lecturers with the tools to implement inquiry-based practice, but to offer practical guidelines that enable them to independently monitor their progress in mastering Inquiry-Based Mathematics Education (IBME) methods and students' engagement with more efficient learning approaches (see Guideline Document available on the PLATINUM website). The preparation of this guide took into account the design, testing, and appropriate instruments that enable an in-depth insight into teaching innovations at local level. Based on the experience of the PLATINUM consortium, case studies will be reported and analysed through a cross-case analysis methodology. We will examine different evaluation and measurement tools which have been used within IBME environments in the Czech Republic, the Netherlands, Spain, and the UK. To support the multi-faceted nature of inquiry-based learning (see Chapter 2) we will critically assess available evaluation tools and criteria and adopt those giving a deeper insight into IBME. We assume that local aims and institutional conditions for IBME activities can vary significantly. Therefore advice on the experience of four national teams will be shared. Investigation on the contexts where the research and evaluation tools were developed will allow us to facilitate the transfer of knowledge to other colleagues from other institutions interested in building and fostering the progress and implementation of IBME in higher education. Our intention in this chapter is not to cover all possible evaluation tools exhaustively. Instead, we would like this chapter to serve as an inspiration for other communities to adjust what they find valuable in the evaluation methods described here to their setting.

Since IBME takes place at several interrelated layers as shown in Figure 9.1 (see also Chapter 2), the selection of possible approaches in this cross-case study can offer insight in the different interrelations of IBME processes from different perspectives. In contrast to traditional evaluation, within the inquiry-based approach the focus is

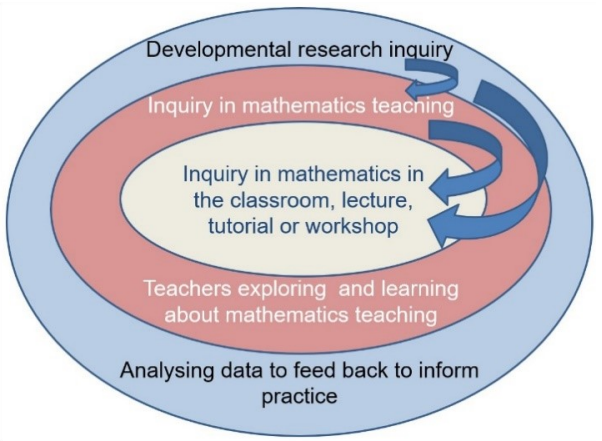

FIGURE 9.1. Interrelated layers in the three-layer model of inquiry.

placed on evaluating the whole learning process covering all layers of the three-layer model of inquiry. Such an approach has both an iterative and a longitudinal nature.

9.2. Research Methodology

A qualitative methodology based on cross-case analysis is used as the methodological research perspective (Borman et al., 2006; Khan & VanWynsberghe, 2008) in order to explore similarities and differences between cases in the PLATINUM consortium.

We purposefully select four cases from different countries (Czech Republic, the Netherlands, Spain, and the United Kingdom) in order to contrast features of the evaluation tools, to implement the methodology of inquiry, and to inform others wishing to adopt it. A number of criteria were taken into account for this choice:

- Different cultural and social contexts.
- The IBME inquiry model consisting of three layers. Inquiry in:
  (a) engaging with mathematics in inquiry-based teaching-learning situations with students;
  (b) exploring teaching processes, the didactic and pedagogies involved in student inquiry, and their use in teaching-learning situations to achieve the desired student outcomes;
  (c) the entire developmental process in which participants reflect on practices in the other two layers, and gather, analyse, and feedback data to inform practice and develop knowledge in practice.
- Some concepts related to IBME such as (1) evaluation of conceptual learning and teaching of mathematics, (2) monitoring students' engagement in IBME, (3) reflection in communities of inquiry on own teaching practice, and (4) professional development of university mathematics lecturers.

This chapter has two dimensions, a theoretical dimension and a practical dimension of design and implementation of evaluation and reflection tools in university teaching practice. The process of preparing this cross-case study has developed in three phases: (1) setting up detailed guidelines for the assessment and evaluation of IBME environments, (2) collecting instruments for evaluation of IBME currently used in the PLATINUM partner universities, and (3) cross-case study analysis of the collected cases.

Multiple cases are taken to establish the range of generality and conditions of applicability of the IBME at university level. The comparative case studies are particularly useful to understand and explain how IBME has been used and which categories have been taken as most relevant. In what follows we will present the case studies in turn. In each of the case studies we will emphasise its characteristics and what contribution is made to further developing the three-layer model of inquiry shown in Figure 9.1.

9.3. Presentation of the Cases

9.3.1. Case in the Czech Republic. At Masaryk University (see also Chapter 13) inquiry-based teaching and learning practices do not have a long tradition. These practices have been implemented in the Mathematics and Statistics I courses in the Faculty of Economics and Administration, and Mathematical Analysis in the Faculty of Education within the PLATINUM Project for the first time.

Our teaching modules are not completely inquiry-based. Inquiry-based activities have a form of small units (either separate tasks or a sequence of linked tasks) incorporated into a traditional curricula. For this reason, the tools used at Masaryk University for courses evaluation cannot be solely applied to IBME as both the traditional procedural approach and the inquiry-based conceptual approach are complementary parts of teaching and learning. Thus, it is not possible to evaluate separately competences achieved solely via inquiry-based tasks. Therefore we decided to evaluate the IBME teaching units individually. The evaluation from the students' perspective has primarily two aims:

- Did students achieve the intended knowledge?
- Were the students active and did they participate actively in the learning process?

The evaluation from the lecturers' perspective mirrors the students' evaluation and follows a similar pattern in asking:

- Were the tasks designed so that they encourage students' thinking and lead to the desired learning objectives?
- Were the tasks designed so that students were engaged and motivated to work on the tasks?

To pursue these aims, we decided to use questionnaires and adopt an experimental design of treatment-control, to observe lectures and seminars, and to organise lecturers' discussion meetings.

Questionnaires. Immediately after the selected seminar or lecture with IBME units, students received a link to a questionnaire with questions related to the benefits of the IBME task from their perspective. The questionnaires combined both questions focusing on learning objectives and questions associated with students' engagement. They had also a space for students' free comments, which proved to be highly beneficial for further development of IBME tasks.

Treatment-Control experiment. The lecturer of two comparable parallel seminar groups on Algebra taught one seminar group with traditional procedural teaching and the other with inquiry-based tasks. The inquiry-based tasks were contained in worksheets that encourage collaboration in small groups. Due to the COVID-19 restrictions and the need to allow students to work in small groups, the ZOOM platform was used as it offers a breakout rooms option and allows an observer to visit and observe students in these virtual rooms. Two weeks later, students from both seminar groups were given the same assessment of the knowledge and skills acquired.

End-of-semester project. In the course Mathematics 2, students completed an inquiry-based end-of-semester team project. The assignment included problems applying linear programming in economics, finance, and management. Teams of three or four students were asked to build a mathematical model of the problem, choose and use appropriate software to solve the problem, interpret the results, and answer additional questions using sensitivity analysis, shadow prices, etc. Groups elaborated the solution independently and met the lecturer every week during office hours to ask for advice on their solution. Finally, students presented the solutions during a lecture slot. After the presentations, the students were sent the feedback questionnaire. Their responses were meant to help lecturers evaluate the activity.

Observations. A qualitative dimension to the evaluation was also added. An observer attended the lectures and seminars where inquiry-based tasks were used and took field notes. Before the COVID-19 related distance learning period, observers were present in class taking field notes to describe the structure of the lessons and the timing of the tasks. Further, the filed notes were complemented by the observers' comments on the students' behaviour, engagement and their inquiry development. During the distance learning period due to the COVID-19 pandemic, the observers were present in the online lessons via the distance learning platform. These observations are used at two levels of inquiry: they help to evaluate (1) students' engagement in the inquiry process and (2) the inquiry-based units from the lecturers' perspective in association to the learning objectives.

One of the observers had a dual role: she was both observer of a seminar group of one of her colleagues and she was teaching the same content in a parallel seminar group. Thus she could utilise her evaluation of inquiry-based activities in her teaching and to share this experience with other colleagues.

Discussion meetings. At the beginning of the PLATINUM project, a Community of Inquiry (CoI) was established, as reported in Chapter 13. One of the purposes of the CoI was to hold meetings to evaluate the IBME units and to discuss further development of IBME units in future. As our community is new to the IBME approach, the evaluation of our experience with IBME units will be valuable for other colleagues at our university. At the meetings, we evaluate many aspects of IBME tasks implemented in our teaching. These include the compliance with the learning objectives and coherence with the traditional curriculum as well as technical aspects such as the timing or the reactions of students who are not used to discussion in the mathematics classroom and do not feel comfortable when risking being wrong when volunteering contributions to the solution of a task.

9.3.2. Case in the Netherlands. At the University of Amsterdam (UvA) the inquiry-based mathematics education (IBME) was implemented in the mathematics courses in the Bachelor Psychobiology and in the Bachelor Biomedical Sciences. A strong community of inquiry has developed around these two courses. The courses had a blended learning design using the digital tools RSTUDIO, SOWISO,[1] and the learning management system CANVAS. During the COVID-19 pandemic the course was online and it used also MS TEAMS. Next to the interactive lectures and tutorial sessions that included short small-group sessions, individual online asynchronous learning activities took place in SOWISO in combination with RSTUDIO. In several parts of the course students worked on realistic problems and discussed solutions following an inquiry based learning approach (see Chapter 12, UvA case study). During the COVID-19

[1]SOWISO is a cloud-based environment specifically designed for learning, practising and assessing in STEM courses, see (Heck, 2017).

pandemic the IBME activities took place in MS Teams channels in groups of 4 to 5 students. The teaching staff team collaborated in the format of a Community of Inquiry via a private MS Teams channel. In all study years the exam was remote and it was taken in SOWISO.

The evaluation of inquiry-based mathematics education (IBME) activities took place on all three levels of the three-layer model shown in Figure 9.1 (cf., Chapter 2): student level, lecturer level, and the developmental research inquiry level. Different instruments were used in this evaluation process. Some of the instruments were developed as part of the PLATINUM project, some were already in use before or they were adapted. The evaluation study was done in two academic years: 2018-2019 and 2020-2021. Henceforth we focus on the evaluation process that was applied to the inquiry-based mathematics module for first-year students in Biomedical Sciences, which is presented in more detail in Chapter 12.

Instruments for students. Two types of instruments were used to evaluate IBME from the student perspective: (a) an online questionnaire and (b) semi-structured small-group interviews. The evaluation instruments for the IBME tasks were integrated into the instruments that were also used for the evaluation of other aspects of the course. The semi-structured interviews were done only in the third year that the Biomedical Sciences course ran.

The online questionnaire used for the evaluation of students' perception of IBME was applied as a pre- and post-test. The questionnaire was administered online in SOWISO. In the pre-test, at the beginning of the course, biographical data and information about the student background were collected. In the post-test the students were also asked to reflect on learning mathematics. The questions in the test were also on mathematics anxiety, test anxiety, and motivation and engagement. Students also took a diagnostic mathematics test. Standard questions sets from standard instruments and translated into Dutch were used to measure mathematics anxiety (Hopko et al., 2003), test anxiety (Spielberger, 1980), and motivation and engagement (Martin, 2007). In the post-test questionnaire, the following three 5-point Likert scale questions were included that were specifically oriented towards the experience with IBME tasks:

- Q1: In some mathematics tasks you had to find out/discover things by yourself. Such an approach of "inquiry-based learning" appeals to me.
- Q2: A small inquiry task to be carried out in pairs for example as a bonus task, seems to me a useful extension of the course.
- Q3: There were sometimes short tasks embedded in the lectures (for example, inventing a method for numerical differentiation and practising with line element fields and direction fields). I learnt much from these tasks.

The semi-structured interviews were conducted after the IBME tasks and took place in the last two weeks of the course before the examination. For this purpose students were invited in small groups of 10. Participation in the interviews was not compulsory but very much encouraged. To get more responses and to lower the generation gap the senior and the junior lecturer were not involved in this stage of the evaluation. The interviewers were the teaching assistants who had attended the course as biomedical students one or two years before and had been involved in the design of the course as members of the CoI. These teaching assistants invited the students for the participation in the interviews. Each semi-structured interview was taken by a team of two teaching assistants; one was asking questions and the other was taking notes. The interviews took place online in MS Teams. The questions in the semi-structured interview were clustered in four groups: (1) similarities and differences with

the secondary school mathematics content and working style, (2) support in working with RSTUDIO, (3) learning materials in SOWISO, and (4) orientation on inquiry-based learning. In the part about inquiry-based learning the students were asked three open-ended questions:

Q1: To what extent does the teaching material encourage thinking about mathematics and its applications in Biomedical Sciences? Did it change your ideas about doing mathematics at all?

Q2: "Having to figure something out for yourself or together with other students" gives a picture of mathematics that does not have to follow a prescribed route or provides no ready-made answers to questions. Mathematics is then seen as a tool to better understand processes or situations and not as a standard procedure to arrive at a correct answer (think, for example, of different regression methods or different techniques for numerical differentiation from which an underpinned selection must be made). Two questions: (a) How new is this to you and how do you feel about it? (b) Do you feel encouraged to do such assignments in the course?

Q3: Would you like to perform more or fewer open-ended assignments and why?

Instruments for lecturers. The procedure for this evaluation level was slightly different for the two academic years in this study. In the study year 2018-2019, the principal lecturers had established an IBME community of inquiry (CoI) together with two junior lecturers. During the course the lecturers met once a week for one hour after the last session of the week with students (face-to-face). Reflective discussions according to the IBME framework and structured oral evaluations were used as instruments during these weekly meetings. The meetings were recorded and minutes were taken. The three lecturers also wrote narratives as their personal reflections. In the study year 2020-2021, the principal lecturer established the IBME CoI together with one junior lecturer and three teaching assistants. The three teaching assistants had followed this module in previous years. The meetings started already four weeks before the start of the course to discuss the course materials and assignments bi-weekly online in MS TEAMS, and weekly when the course had started. The teaching assistants reflected on their own learning experience as students and the team members collaborated on the development/adaptation of the (new) materials using their reflections.

The questions for the semi-structured interviews were developed by the teaching assistants in collaboration with the lecturers. The pairs of teaching assistants who did the interviews wrote a report of each interview and shared it in the IBME CoI. The results of the students' questionnaire (pre- and post-test) and the analysis of the reports of the semi-structured interviews are presented and discussed in the UvA case study in Chapter 12 of this book.

Instruments for developmental research inquiry. One PLATINUM project team member joined to the lecturers' Community of Inquiry of the Biomedical Sciences course. She attended the meetings of this CoI as an observer of the process on the level of the developmental research inquiry. She observed also a lecture given by the senior lecturer in this CoI in which a short IBME task was used, and a group IBME session where students worked on a longer IBME task based on biomedical research data and the programming language R. The instrument on this level was making observation notes and writing narratives based on observation notes.

9.3.3. Case in Spain. At the Complutense University of Madrid (UCM) inquiry-based mathematics education (IBME) is implemented in the mathematics courses in

the Bachelor Mathematics, Bachelor Mathematics and Engineering, Bachelor Mathematics and Statistics, Bachelor Computer Engineering, and in Programmes of Professional Development for mathematics lecturers. The evaluation of inquiry-based activities took place on three levels: (1) the student level, (2) the lecturer level, and (3) the level of lecturers' professional development. Different instruments were used in this process. This section focuses on the lecturers' professional development. The case presented is about the professional development of novice lecturers, in particular within the training unit about teaching Rolle's Theorem: "Intuition on Rolle's Theorem and its extensions." The materials and evaluation instruments are original and specifically developed for the PLATINUM project.

Professional development and teaching context. The Faculty of Mathematics at UCM develops courses for university teaching qualification of novice lecturers. These courses aim to provide university lecturers and research assistants with educational tools that enable them to better design, implement, and analyse teaching and learning processes. Three organising principles guide the design of these resources:

- To enable lecturers and research assistants to make informed decisions on what they teach and how they teach it.
- To train novice lecturers and research assistants who are becoming lecturers, in the growth of their conscious awareness: self-awareness as lecturer, awareness of discipline, awareness in guiding others by teaching them to learn, and by learning to learn.
- To develop lecturers' professional identity through a continuous reflection on their professional role and their specific vocation.

All novice lecturers participating in the PLATINUM professional development course on inquiry-based mathematics education had to design mathematical tasks or units following the inquiry approach to be implemented in the classroom. These tasks were presented at team meetings and discussed together before they were implemented. The lecturers were also observed during the teaching of a mathematics unit (this means between one and three lectures/sessions) and the sessions were video recorded. The PLATINUM community of inquiry gave feedback, watched the video of the lecture, and reflected on the teaching, the behaviour of students during the lecture and the anticipation of the lecturer on the activity of the students and evaluated these.

In the proposed conception, research and development are mutually involved. Professional development is viewed from a reflexive position concerning practice. The aim is for the novice lecturers to join in the PLATINUM project over the practice, questioning and analysing it, and even transform it according the approach of inquiry-based mathematics education.

It is important to remark that there exist two types of context: (1) the professional development course or formative situation, involving the trainer (professor in mathematics education) and trainees (novice lecturers), and (2) the teaching situation in which the lecturers work with undergraduate students. These situations produce different levels of activity and practice for professional development and the lecturers-in-formation (see Figure 9.2).

Different instruments are used in this evaluative reflection process: (1) the lecture plan and the proposal for the inquiry based tasks, the planned teaching, learning and (formative) assessment activities in the time frame of the session; (2) the video recording of the lecture/session; (3) the peer feedback, the observation report of the observer of the session (in the case presented here, the lesson was observed by two members of PLATINUM, one from Spain and one from England); (4) the students' evaluation

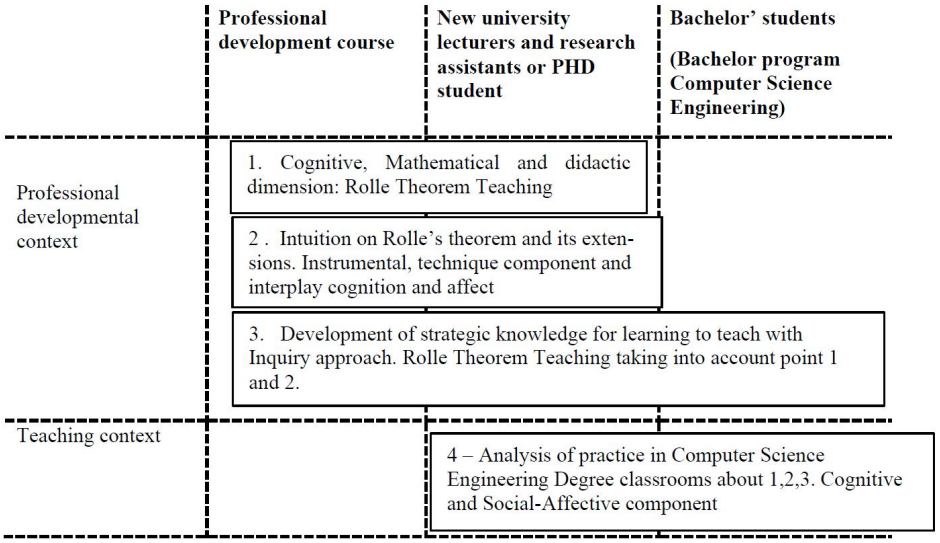

FIGURE 9.2. Professional development and teaching context at UCM.

(questionnaire); (5) the novice lecturer's evaluation (semi-open questionnaire) and interview; and (6) the reflective report of the novice lecturer based on self-observation, peer-observation, and the students' evaluation.

Here we consider two evaluation instruments: (1) a student evaluation questionnaire, and (2) a questionnaire for new lecturers' evaluation.

Instruments of evaluation for professional development — Teaching Rolle's Theorem. Prior to the presentation of the evaluation instruments, we present the learning tasks on the lesson "Rolle's Theorem teaching: Intuition on Rolle's Theorem and its extensions."

Overview plan of inquiry-based tasks. Calculus is a first year subject of the bachelor program in computer science engineering. It is an introductory course that starts with the definition of real numbers and the construction of sequences and series, and it covers differentiation and integration of functions of one variable with applications. Weeks 9 to 11 of the course focus on functions and their derivatives. The main concepts regarding this subject had been already learned in high school, but since the backgrounds of the students vary greatly a revision and reconstruction of some these ideas is deemed necessary. Rolle's theorem and its extensions (the Mean Value Theorem) is one of the big theorems in Calculus because it establishes a connection between continuity and differentiability. Though it is a very simple and intuitive theorem, it requires the understanding of limits, continuity, and differentiability. The goal in the mathematics unit is to clarify different concepts regarding functions and to redefine them with intuition. The inquiry-based tasks are articulated in:

(1) A revision of the known concepts: who is who?
(2) Intuition on Rolle's Theorem and its extensions.
(3) Understanding the concept of derivative. For more information see (Luque, 2019).

Some instruments. Two evaluation tools are described below.

(1) *The student questionnaire.* The questionnaire was administered to a group of 34 students enrolled in the Bachelor's Computer Science Engineering after receiving

theoretical and practical background on the concept of functions and derivability.[2] The questionnaire was divided into three parts:

(i) Functions and derivatives concepts through the inquiry process.
(ii) Rolle's Theorem: mental image and intuitive understanding. Students were asked to explain in their own words Rolle's theorem and to identify in different plots of graphs whether it is possible to apply the theorem.
(iii) Grading the understanding and interplay between concepts continuity and differentiability: metacognitive and affective factors can inhibit the correct utilization of students' knowledge.

(2) *The novice lecturer questionnaire.* At UCM the PLATINUM evaluation tools have been elaborated within the Design-Based Research Collective (2003). Two hypothesised dimensions that constitute mathematical knowledge for teaching were the focus: mathematics content knowledge and mathematics pedagogical content knowledge. In identifying the elements of these dimensions, the design of inquiry-based tasks and the development of these tasks in the classroom (Figure 9.2) are considered for the evaluation instrument for novice lecturers.

The questionnaire is intended to ensure that the novice lecturers' practice is based on self-observation and the students' evaluation, and that novice lecturers reflect on their teaching practice and the strategic knowledge that they have developed for teaching. The semi-open questionnaire is structured into three parts:

(i) Regarding inquiry-based tasks design: it was inquired at three moments, viz., before, during, and after the implementation time.
(ii) Regarding mathematics conceptual topics: intuition on Rolle's Theorem and its extensions and interplay between concepts. The teaching plan aimed to balance all three components of mathematical representation (graphical, numerical, and algebraic) to enable the students to view ideas from different standpoints and develop their intuition and a holistic perspective of each concept. It allows to review the results of the students in the sense of (i) the learner's ability to state the theorem and apply it to reasoning tasks, (ii) the influence of concept images in his or her reasoning about the theorem, (iii) the learner's ability to perceive the relationship between Rolle's Theorem and other related mathematical concepts and mathematical attitude, and (iv) metacognitive and affective factors possibly inhibiting the correct utilisation of knowledge that the students should use to solve a problem. We note that this block of questions in the questionnaire cannot be seen in isolation; it is in close relation to the questions posed to students in their questionnaire.
(iii) Regarding the mediation of the PLATINUM Community of Inquiry. This group has had a significant influence on the lecturer's professional development, offering teaching intuition. In this section of the questionnaire there is a reflection on the tacit knowledge dimension acquired. Some aspects considered are the following:
    - goal setting in motivating and guiding oneself to attain the desired end goal;
    - systematic problem solving by using resources and orientations;
    - personalising situations by appropriating one's strengths and weaknesses.

    They are asked to develop a narrative of their experience.

9.3.4. Case in the United Kingdom. The type of evaluation described in this case differs from the previous cases. It involved reflecting on the work of small, informal groups of mathematics and mathematics education lecturers coming together

[2]See the PLATINUM website: `https://platinum.uia.no` and (Gómez-Chacón & Luque, 2019).

to talk about their teaching practice and the connected educational research. These meetings were not part of a training organised by the institution, but they did originate from the desire to discuss mathematics-specific teaching and learning issues. As described in Chapter 15, Loughborough University (LU) in the UK has a long tradition of formal and informal collaboration between mathematicians and mathematics educators in Communities of Inquiry (CoIs). These CoIs can take the form of small groups of colleagues reflecting about teaching and learning mathematics, or are supported via funded projects that aim at involving students in the creation and testing of inquiry tasks. In this chapter, the evaluation process of one informal CoI involving mathematics lecturers and mathematics educators colleagues is summarised.

Some ideas about evaluation. What we discuss in this section is not and cannot be a rigorous evaluation of the impact of taking part in a CoIs on the participants' teaching practices. What are described here are some ideas that can help colleagues trace some of the outcomes of small, often informal, reflection both on the practice and on educational research for those who have taken part in the CoI. The main inspiration for these principles is the work that Pawson and Tilley (1997) report on realistic evaluation. The guiding principle of realistic evaluation can be summarised by the following quote:

> Whereas the question which was asked in traditional experimentation was, "Does this work?" or "What works?", the question asked by us in realistic evaluation is "What works for whom in what circumstances?" (Tilley, 2000, p. 4)

Of course, to follow the principle of realistic evaluation for large interventions (not only educational) is complex and requires a well-structured team of researchers versed both in quantitative and qualitative research. However, from this work we can find three guiding principles that can be useful also for more informal qualitative evaluations of smaller activities. These principles are:

- Focus on the context where the evaluation was introduced. What are the contextual characteristics that may lead one activity to be successful in one implementation and not in another?
- Small evaluations can ask bold questions regarding in our case the effectiveness of informal CoIs. Pawson and Tilley (2001) argue in one of their writings that even very small informal interventions, if guided by theory, can contribute to the refinement of that same theory.
- It is important to focus not only on the outcome of the intervention (did participation to a CoI of colleagues discussing teaching and learning change the practices of those who took part in it?) but also on the mechanism that lead to such intervention. The description of such mechanism will help others ascertain whether that intervention has the potential to be successful in their own context.

In what follows we discuss how these principles have guided the evaluation of one of the case studies that took place a Loughborough University.

The teaching group: a small informal CoI. As described in Chapter 15 this was the work of a small group of mathematicians and mathematics educators (all teaching mathematics or statistics at university in the same institution) who met four or five times per year for three years to discuss topics related to the teaching and learning of mathematics at university level. The details and general aims of these meetings are described in Chapter 15. Here we want to focus first of all on what was evaluated, and on—paraphrasing Tilley (2000)—what worked for whom in what circumstances. The questions we asked were:

- What activities were conducive to effective reflection on practice?

- What activities had a visible impact on practice?
- What activities were not deemed to be beneficial?
- What were the aspects that facilitated participation to the sessions?
- What were the aspects that prevented participation to the sessions?
- What were the contextual factors that facilitated (or prevented) participation to the sessions?

Given the nature of the questions and of the activity that is investigated we collected two distinct types of data: the documentations discussed throughout the existence of the group meetings and a series of semi-structured interviews with stakeholders. The documentations consisted in research papers that the participants suggested as reading, materials related to teaching that were brought to be shared (e.g., questions in exam papers, or suggestions for feedback to students), or simply questions for discussion by the group. Analysis of the documentation collected indicates that one of the main concerns of the group was summative assessment practices in mathematics. This reflects a general preoccupation in the institution where the CoI was based and in the UK more at large with issues related to assessment. During these sessions we would both discuss concrete examples of exam questions volunteered by one of the participants and research papers on the topic. The analysis of such documents is very important on this evaluation as it allows the analysis of the contextual factor that guided the interest of the CoI.

Regarding the semi-structured interviews, stakeholders are considered to be not only colleagues who took part in the meetings of the teaching group but also those with responsibility in the mathematics department connected to teaching. Therefore in the case of the Teaching Group, stakeholders were not only those who took part in the session of the CoI but also those with responsibility for teaching in the mathematics department, such as the head of the department and colleagues who had shown an interest in teaching and learning of mathematics (from our experience) but did not take part in the sessions of the CoI. Interview questions included in the interview schedule were:

- When you started at Loughborough University, were you new to teaching mathematics at university level? What kind of students/year groups have you taught or are you teaching now?
- What aspect of your teaching are you particularly pleased with, or alternatively, are thinking of changing?
- Consider a course that you have taught more than once. Did you make any changes from one year to the next? Why?
- Do you think you teach like your colleagues?

These questions aimed to investigate from the general to the particular and aimed to ascertain participants' perceptions of the benefits or drawbacks of having a CoI like the Teaching Group in the department. Through the analysis of the interviews it was possible to understand the trajectory of the CoI, which stopped meeting in July 2019. The analysis of the data also allowed us to understand the role of the 'value' that was put on such initiatives by the Institution and the fact that without even informal institutional support such activities cannot flourish.

9.4. Contribution of the Cross-Cases Study: Challenges and Issues

The choice of the cross-case study methods was made to highlight the differences that can occur in the implementation of Inquiry-Based Mathematics Education (IBME) in different contexts and at different layers of the theoretical model. Therefore

there is a range of foci of evaluation and evaluation tools. The cases described in the chapter intend to be an example of foci and tools that others can follow when evaluation their own IBME in their own context. In this section we synthesise commonalities and differences in the evaluation using IBME in four contexts and to what extent they become challenges for future implementations.

A first observation is how each partner contributed to the three-layer model of inquiry and the notion of a Community of Inquiry (CoI). Since local aims and institutional conditions for IBME activities can vary significantly, advice on the experience of seven national teams will be shared. The cases presented here contribute to enriching the layers and the interplay between layers shown in Figure 9.1). For instance, the selection made by Loughborough University focused on Communities of Enquiry (CoIs) between mathematicians and mathematics educators and offered ideas for the evaluation process of small CoIs. All universities cover all three layers of the three-layer model of inquiry; however, each university emphasises its specific area. The University of Amsterdam takes into account instruments for students and lecturers implemented in the mathematics courses in the Bachelor Psychobiology and Bachelor Biomedical Sciences, and Masaryk University with instruments implemented on the courses Mathematics and Statistics I at the Faculty of Economics and Administration, and Mathematical Analysis at the Faculty of Education.

The case of the Complutense University of Madrid focuses on the professional development of mathematics lecturers and offers an insight into the different interrelations between layers. It also proposes tools to evaluate the entire developmental process in which novice lecturers react on practices in the other two layers (teaching in the classroom and receiving feedback data to inform their practice and develop knowledge in practice).

In the analysis of the differences in the cross-case analysis, we highlight two: (1) the characterisation of the inquiry community and (2) how the choice and integration of various tools for evaluation has taken place.

Communities of inquiry. To support the multi-faceted nature of inquiry-based learning showcased by the PLATINUM project and presented in the intellectual output about evaluation, a key element has been the type of community of inquiry. Some highlights are the previous trajectory in mathematics education or the member composition (mathematicians members-only, or a mixed community of mathematical educators and mathematicians). For instance, the Czech Republic team's expertise in statistics and statistics education, and in the nature of the IBME has enabled them to trial an experimental design and a more quantitative approach. The LU experience focuses on the realistic evaluation approach, and the UvA and UCM cases combine a natural approach with design-based research that includes different cycles of monitoring and evaluation.

Choice and integration of various tools for evaluation. The evaluation tools used in each case reflect the nature of the CoI and the activities evaluated. Each team critically assessed the available evaluation tools and criteria and adopted those to their context giving a deeper insight into the working of their IBME. This chapter aims at equipping the readers with similar tools to critically evaluate tools that allow them to adopt what is most suitable to the situation investigated.

9.5. Conclusions

We have presented four implementations of IBME in the teaching practice. In the experience of evaluation of very different CoIs we highlighted how each one contributed to developing further the three-layer model of inquiry that the PLATINUM

CoI adopted (Figure 9.1, see also Chapter 2). We have deliberately sought to compare cases that differ in their forms of evaluation implementation in order to find similar processes or outcomes in the IBME PLATINUM approach. We believe that this case-oriented approach emphasises diversity in the selection of cases. Its potential lies in its ability to extend lessons learned in individual cases to inform another case and discover similar processes in unexpected contexts. In examining the differences between the cases we have covered both learning about mathematics with students and learning about teaching and learning mathematics with lecturers. The common aspect that the cases presented have is the engagement in the CoI and the subsequent engagement in developing the work of the CoI in the light of what was learned through the process. This—as we have seen previously—is the third layer of the CoI and the one that needs developing in time.

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