Inquiry in University Mathematics Teaching and Learning: The PLATINUM Project
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CHAPTER 3

Spidercharts: A Tool for Describing and Reflecting IBME Activities

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3.1. Introduction

The main goal of PLATINUM was to strengthen IBME in university mathematics teaching within a collaborating network of European partner universities, which saw itself as a *Community of Inquiry* (CoI) (see Section 2.5). As the visions of IBME present among project partners as well as local conditions at the partner universities were very diverse, efforts were made to account for local specificities in the development and implementation of ideas and concepts of IBME—in accordance with the notion of CoI. In this process of community building, its diversity became a fruitful resource, which enriched interactions at jointly conducted workshops (cf., Chapter 7). A key factor for successful cooperation within the community was good project communication.

Successful project communication relies on participants' familiarity with basic terms and concepts of relevance to project goals, and on their ability to identify and communicate parallels and differences between the many views that are present among partners. This enables project partners to locate their respective objectives and local conditions within the shared project context and to relate them to each other. With this in mind, we developed three spidercharts as a tool to facilitate project-wide thinking and communication about activities of local groups and to promote reflection on and further elaboration of the common vision to integrate IBME in our teaching. The spidercharts were developed both theory-based and inductively. We will say more about this in the next section.

This chapter is structured as follows: In the next section, we first describe theoretical considerations that inspired the development of the spidercharts. In Section 3.3, we explain the meaning of the labels in the spidercharts. After this, we present in Section 3.4 the results we obtained from each partner when we asked them to fill in the three charts with respect to the specific PLATINUM case they present in their respective chapter. Lastly, we shortly describe some patterns and distinctive features that became visible when comparing our partners' completed charts.

3.2. Developing the Spidercharts

We came across the idea of spidercharts in a contribution by Lübcke, Reinmann, and Heudorfer (2017). The double-wheel model presented there was used in a large research project to classify research-oriented teaching at university level into types. The double-wheel model is a further development of the wheel model by Brew (2013) that was created to facilitate "the identification of choices to be made in developing research-based pedagogies" (p. 612). Both, the wheel and the double-wheel model

subsume curriculum design under pedagogic choices and propose dimensions for reflection on pedagogic choices in inquiry-oriented teaching settings. The double-wheel model by Lübcke et al. (2017) differentiates between a *micro-level* that focuses on didactical choices and a *meso-level* that has a stronger focus on curricular considerations. Neither model focuses explicitly on mathematics education, but both are specific to university contexts. We borrowed the spiderchart idea with the intention of designing a similar model as a reflection tool for the PLATINUM community. For this, we had to adapt the model to the specific needs and theoretical basis of the PLATINUM project. Specifically, our model needed to reflect important general dimensions of IBME. Here, we had to strike the following balance in our design: on the one hand, we wanted to take into account the wide variety of IBME approaches and activities present in PLATINUM; on the other hand, we wished to maintain a certain degree of specificity, in the sense that the charts' collection of aspects proposed for reflection should have specific relevance to mathematics as a university subject. In the following we will explain in more detail the intentions and ideas that guided the development and design of the spidercharts.

The spidercharts are intended as a supportive tool for (groups of) people who engage in the development of IBME activities and their implementation in teaching-learning environments (e.g., lecturers, teacher-researchers; henceforth *teachers*).[1] They are meant to guide teachers through a structured session of reflection about their teaching—or, more precisely, about one of their IBME activities. This guided reflection can spark new ideas that help flesh out the notion of IBME underlying the teaching project and can create opportunities to uncover formerly overlooked potentials in the local teaching-learning setting. Inspired by the theoretical context of our PLATINUM project, we decided to create three charts that address three perspectives on a teaching-learning scenario that are of major importance in the development of IBME activities: the perspective of students and their scope of possible activities within the learning environment is addressed in the chart *inquiry learning*; the perspective of teachers when working in the classroom is captured in the chart *inquiry teaching*; and the perspective of teachers in their work environment outside the classroom is the topic of the third chart, *group of inquiry*. In other words, each spiderchart proposes as focus of reflection a set of aspects relevant to the following questions concerning the processes and decisions involved in organising IBME activities:

- Chart *inquiry learning*: What possibilities of engaging with the IBME activity are available to students?
- Chart *inquiry teaching*: What methods of conducting an IBME activity are realised by the teacher? What restrictions are present in the specific learning scenario?[2]
- Chart *group of inquiry*: What structures and dynamics are at play in the group's work to support and reflect the IBME teaching of their members?

The questions focus mainly on methodical-didactical design elements in the development and implementation of IBME activities: At the centre of attention of the charts *inquiry learning* and *inquiry teaching* is not the mathematical subject matter that is the object of the inquiry activity unfolding in the classroom, but the spectrum and quality of opportunities for engagement with the (mathematical) object of inquiry that the teaching-learning environment opens up to students (and teachers). Thus,

[1]This can but does not necessarily include researchers who research the implementation of IBME activities.

[2]Such restrictions might be set by the teacher or be imposed on students and teacher alike by the institutional context.

these two charts draw attention to the types of activities that are allowed or expected to be conducted (in the classroom) in order to approach and investigate the (mathematical) subject matter at hand. Filling out these two charts leads to a visual display of didactical choices made with the intention to engage students in (mathematical) inquiry. Activities and didactical choices may involve tasks, a specific setting, curriculum design, considerations in relation to the student group, the culture, etc. To what extent it is possible for teachers to integrate IBME into their regular teaching (e.g., constructing a whole module, creating one session within a course) depends of course on local conditions. The third chart *group of inquiry* similarly addresses methodical aspects, but this time of the teachers' work environment. It focuses on the collaborative work of the local group of inquiry and serves to reflect and visually display how the local communities within PLATINUM organise their teamwork. In particular, it calls to attention teachers' possibilities within their professional environment to engage in inquiry into their IBME teaching.

With these key questions and ideas in mind, we will now elaborate in more detail on the aspects and quality nuances addressed by the spidercharts: Each chart covers eight *aspects* of IBME that may be relevant to the development of IBME activities within the local group and their implementation in the classroom. Aspects can have different qualities in different teaching-learning settings. This is expressed by the *quality nuances* assigned to each aspect. The aspects and their respective quality nuances displayed on the spidercharts mirror basic principles of inquiry-based education (Artigue & Blomhøj, 2013; Dorier & Maaß, 2020; Lübcke et al., 2017) and also address collaborative developmental work within communities.

For instance, Dorier and Maaß (2020) characterise inquiry-based mathematics education (IBME) as follows:

> *Inquiry-based mathematics education* (IBME) refers to a student-centered paradigm of teaching mathematics and science, in which students are invited to work in ways similar to how mathematicians and scientists work. This means they have to observe phenomena, ask questions, look for mathematical and scientific ways of how to answer these questions (like carrying out experiments, systematically controlling variables, drawing diagrams, calculating, looking for patterns and relationships, and making conjectures and generalizations), interpret and evaluate their solutions, and communicate and discuss their solutions effectively. (p. 384)

The quotation formulates an idealised vision of teaching and learning. Although it is mainly learning that is explicitly described, the word "invite" indicates a vision of teaching whose specifics are not elaborated in the quotation, but which is nevertheless indirectly indicated by its ambition.

In addition, Artigue and Blomhøj (2013) provide an overview of existing forms of IBME with a strong focus on approaches typically pursued in school contexts. They present a broad variety of different understandings of IBME and their relation to different constructs, institutional contexts and educational cultures. In summary, they point out different prioritisations in the context of IBME of the following concerns:

[1] the 'authenticity' of inquiry questions, the connection of students' activities with their real life, links between everyday-life questions and activities;
[2] the epistemological relevance of inquiry questions from a mathematical perspective and the cumulative dimension of mathematics;
[3] the progression of knowledge as expressed in the curriculum;
[4] extra-mathematical questions and the modelling dimension of the inquiry process;
[5] the experimental dimension of mathematics;

[6] the development of problem-solving abilities and inquiry habits of mind;
[7] the autonomy and responsibility given to students, from the formulation of questions to the production and validation of answers;
[8] the guiding role of teachers' and teacher-students' dialogic interactions;
[9] the collaborative dimension of the inquiry process;
[10] the critical and democratic dimensions of IBME.

To illustrate how these concerns contributed to the spidercharts' development, we will now present an exemplary collection of connections between them and concrete aspects and quality nuances displayed on the spidercharts. We will do this by explicitly stating possible questions and thoughts for reflection that arise from the respective concern and naming the aspect(s) or quality nuance(s) connected to this question:

In what way is the connection between mathematical knowledge and real life relevant? What relevance does the modelling dimension[3] of the tasks we provide have (see [1] and [4])? Do teachers in their teaching focus on *applied* or *theoretical mathematics*? [chart *inquiry teaching*; aspect: *type of mathematics*]

If mathematics is taught with a stronger emphasis either on application or on formal/scientific criteria, is this a didactical choice made by the teacher or is this situation predetermined by the curriculum? [This is an underlying concern of the spiderchart *inquiry teaching* that should be kept in mind when using it.]

What is the epistemological relevance of the questions or the content that we address in class from a mathematical point of view? And what importance do the questions or does the content have for the cumulative dimension of mathematics (see [2] and [3])? Is the manner in which asking questions, communication of findings and justification take place in class open to student choice, or determined by the teacher or other factors (i.e., closed)? Is the form of justification open for choice or is it required to follow formal/scientific criteria? [chart *inquiry learning*; aspects: *asking questions*, *communication of findings*, *justifying*]

The experimental dimension of mathematics points to similarities between inquiry learning in mathematics and inquiry-based education in other disciplines (see [5]). We highlight the importance of making observations, asking questions, planning investigations and using tools within an inquiry process in the spiderchart *inquiry learning*. [chart *inquiry learning*; aspects: *making observations*, *asking questions*, *planning investigations*, *using tools*]

What degree of autonomy and responsibility is given to and demanded from students (see [7] and [8])? This question motivated the definition of certain quality nuances we attached to the charts' aspects (esp. in the chart *inquiry learning*). If the didactical choices grant students a high degree of autonomy and responsibility, the quality nuance assigned to the aspect in question (e.g., aspects: *asking questions*, *communication of findings*) is open. If a low degree of autonomy and responsibility is granted to students then the quality nuance of an aspect is closed. Regarding the quality nuances open and closed, it should be kept in mind that the autonomy and responsibility given to students bears implications for the role of the teacher in the respective teaching-learning environment. [This is a general concern connected with the spiderchart *inquiry learning*.]

[3]For the development of modelling tasks in PLATINUM, see Chapter 8.

So far we have described how we have selected aspects and quality nuances based on theory. Additionally, the development of the spidercharts was done inductively in that the concrete selection of aspects was also based on discussions at our international PLATINUM meetings (discussion notes, feedback by PLATINUM partners): We repeatedly asked partners about their local cases at different stages of their development. With the help of this information we checked whether the categories of the spidercharts and their characteristics could be interpreted in a meaningful way and whether, from our and our partners' point of view, essential aspects of local cases could be addressed. In the workshop in Brno we could observe that the spidercharts could be successfully used as a tool for reflection. When they were filled in, they triggered a wide range of discussions about the current design of the cases and about the possibilities for further adaptations. However, the Brno workshop also produced incentives to revise some of the categories. After this revision, we approached the partners a second time with the updated spidercharts and filled them in together with them.

All in all, our efforts yielded the following eight aspects for each of the spidercharts:

- *Inquiry learning:* exploration, planning investigations, communication of findings, justifying, asking questions, cooperation, using tools, and making observations.
- *Inquiry teaching:* assessment format, media, type of mathematics, content, teaching methods, tasks, scaffolding, and feedback.
- *Group of inquiry:* deciding on objectives, access to group, discussions, evaluation, reflection on professional growth, reflection on teaching, joint planning of teaching, and organisation of group work.

Moreover, each of these aspects has specific quality nuances assigned to it. Some quality nuances appear in connection with a number of different aspects, some appear in connection with only one single aspect. The different quality nuances that appear are:

- open & closed/& standardised/& structured/& formal/scientific
- essential & non-essential
- formative & summative
- weakly formatted & strongly formatted
- reduced facilitation & increased facilitation
- student-centred & teacher-centred
- student-chosen & teacher-chosen
- digital & analogue
- applied & theoretical

Each quality nuance is intended to represent a continuum of possibilities. Therefore, we additionally introduced the two middle options of *quantitative* ("quan.") and *qualitative* ("qual."): *quantitative* expresses that with regard to all occurrences of an aspect (in an IBME activity), each end of its quality nuance applies to (roughly) equally many of these occurrences; *qualitative* expresses that with regard to the occurrences of an aspect, both ends of its quality nuance are equally important, in the sense that most occurrences of this aspect are located between both quality nuance ends in terms of gradation.

In the following section, we explain the use of the spidercharts and provide detailed descriptions of each chart's aspects and their respective quality nuances. Afterwards we present results of a survey conducted with the spidercharts among the partners of the PLATINUM project.

3.3. The Three Spidercharts and How to Work With Them

Because the spidercharts are intended as a tool for reflecting IBME activities, the first thing to do in order to use them is to decide, which IBME activity should be focused and what exactly constitutes this activity. Additionally, the group of inquiry relevant for the third spiderchart needs to be specified. These choices impact the significance of each of the three spidercharts for the description and reflection of the chosen IBME activity. The spectrum of eligible IBME activities is broad and can range from a single exercise conducted in one specific session of a course to an entire course which was designed to contain IBME. Instead of a course it is also possible to pick a course plan which has not yet been implemented, if it is sufficiently detailed. The relevant criterion for selecting the group of inquiry is the group's participation in the development, implementation or research of the activity in question. The group could for example be a group of researchers who developed an IBME activity or a team of teachers who work together to promote IBME in their courses.

After this first step has been completed, the charts can be consulted. If we take a quick look at the charts on the upcoming pages, we can see that the central label of each spiderchart shows the perspective the chart addresses: *inquiry learning*, *inquiry teaching*, or *group of inquiry*. The outermost ring of each chart shows the *aspects* of the respective perspective. The three rings between the centre of the chart and the outermost ring are divided into fields that will be ticked in the process of filling in the chart. The four fields between each aspect and the chart's centre name the *quality nuances* of the respective aspect (cf., for example, in the chart *inquiry learning*, the quality nuances "open" and "closed" of the aspect *exploration*, and their middle options "quan." and "qual.").

In order to characterise a specific IBME activity and its context using the spidercharts, we ask the user to tick one quality nuance field for every aspect with the selected IBME setting in mind: Of the four fields for every aspect, ticking the inner- or outermost one means, that the descriptor in the respective field describes that aspect of the chosen setting best. The middle should be ticked, if the descriptors on the two ends of the quality continuum both apply equally strongly in the context in question. In this case, a choice must be made between ticking the field "quan." (for quantitative) or the field "qual." (for qualitative). The rule of thumb is that "quan." should be ticked if the decision to tick the middle was taken because the teaching-learning setting that is being reflected with the help of the charts hosts a quantitative mix of features or elements of which some are best described by the innermost field and others best described by the outermost field. The field "qual." should be ticked, if the decision to tick the middle was taken because both inner- and outermost fields are qualitatively equally important or applicable to the feature(s) of the user's setting referred to by the respective aspect (i.e., if said features are really located in the middle of the continuum defined by the two quality nuances at its two ends).

The aspects in each spiderchart may overlap in meaning and may also be correlated with each other. This can be more or less the case depending on the specific scenario chosen to be described or reflected with the help of the spidercharts. However, this does not pose a problem to the central goal of the activity of filling in the three charts, which is to bring to the attention of the user different aspects which are relevant to IBME activities, to inspire thought about their nature in the context of the user's own IBME project and to start a conversation about a specific IBME activity among the members of a group of inquiry. Furthermore, the aspects of the spidercharts are generally not intended to be interpreted strictly within the context of mathematics as a subject.

For example, *exploration* can mean any type of exploration, it is not restricted to designate a purely mathematical activity. Some aspects might have different meanings or are likely to be interpreted differently in different subjects or faculties. Which interpretation is relevant for the IBME activity to be reflected is up to the members of the involved group of inquiry.

With a view to the application of the three spidercharts in practice, we now formulate their foci as action-oriented as possible and explain the meanings of all aspects. Afterwards we look at the quality nuances and illustrate them by examples.

Spiderchart inquiry learning. This spiderchart focuses on aspects which are relevant for IBME from the perspective of the learner. Filling in the chart can be facilitated by keeping the following questions in mind:

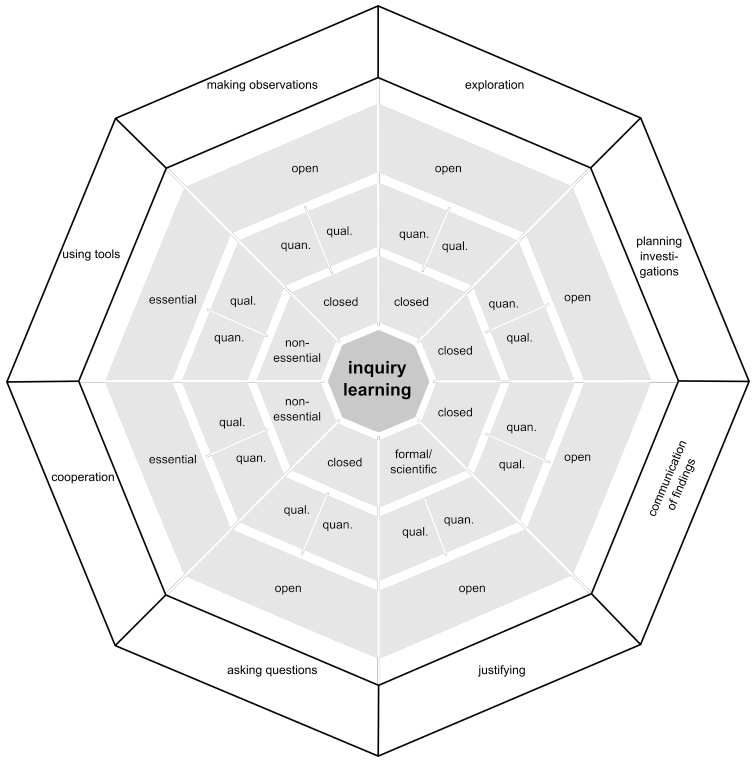
- What do students need to do in order to participate in my IBME activity?
- In what ways are students engaged in inquiry learning?

Spiderchart inquiry teaching. This spiderchart brings to attention aspects of the instructional setting which may or may not be under the control of the teaching staff responsible for the chosen IBME activity. The following questions might help with filling in the chart:

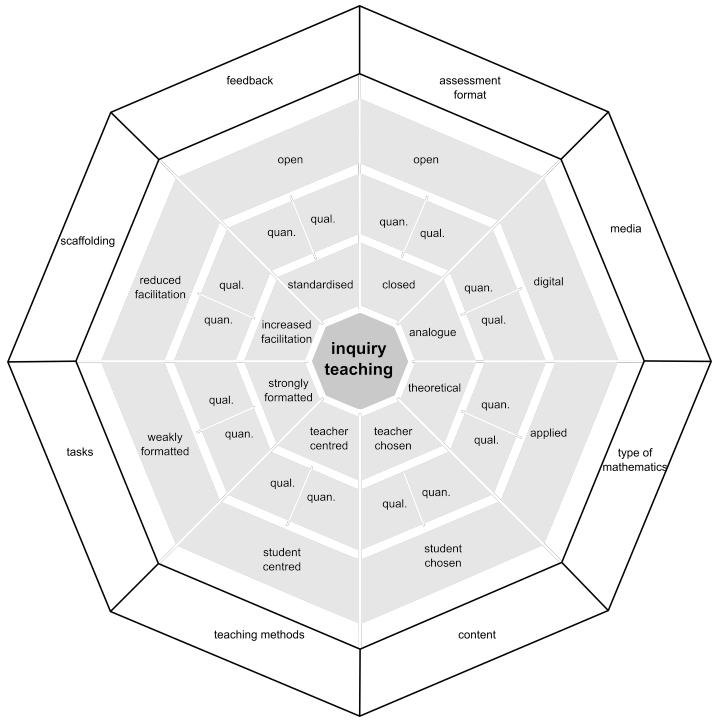
- What did the instructional setting of my IBME activity look like?
- What does the envisioned instructional setting of my IBME activity look like?

Spiderchart group of inquiry. This spiderchart is intended to help describe the collaboration in the group of inquiry who develops, teaches and/or does research on the chosen IBME activity. The following questions might help bring to mind information relevant to filling in this chart:

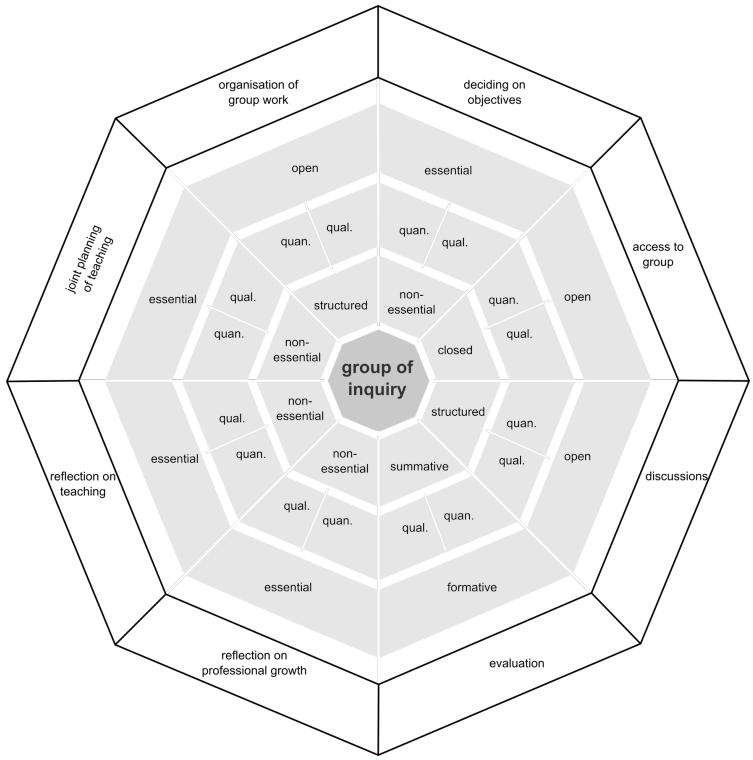
- How does my group of inquiry work on projects and tasks?
- How is collaboration in our group organised?



Aspect	Definition
exploration	Learners engage in exploratory activities in view of an unfamiliar problem/mathematical phenomenom/etc.
planning investigations	Learners plan a structured investigation into a (larger/more complex) mathematical topic or problem.
communication of findings	The manner in which learners communicate results of investigations or thought processes to peers, the teacher or other people. They might talk informally, write a proof, hold a presentation, etc.
justifying	Learners give justifications for statements or choices they made. Justifications can be mathematical, scientific, common sense, naive, etc.
asking questions	Learners ask questions at different stages of an inquiry process: to initialise an inquiry, to refine a question, to question findings, etc.
cooperation	Learners form groups and interact in order to jointly work on an IBME activity.
using tools	Learners make use of digital tools, algorithms, specific heuristics or strategies, etc. as a means to achieve some goal (e.g., representing or depicting something, solve a task, explore a phenomenon, etc.).
making observations	Learners make and possibly articulate observations about some phenomenon of interest or some problem situation.



Aspect	Definition
assessment format	Any mechanism of evaluation of student performance, knowledge or skills. This may but does not have to produce a grade.
media	The means of communication (e.g., text on paper, video, speech, whiteboard, etc.).
type of mathematics	We distinguish between pure or theoretical mathematics (e.g., proofs, logic) and applied mathematics (modelling, mathematics used in other subjects).
content	The knowledge, topics and questions of the IBME activity.
teaching methods	The intended ways of knowledge creation and exchange that are supported/managed/implemented by the teacher.
tasks	Exercises and problems given to learners in the context of an IBME activity in order to structure the educational process. Learners are typically expected to provide some sort of task solution as a result of working on the task.
scaffolding	Additional hints, instructions or information that accompany a given task in order to facilitate the process of solving it (e.g., provision of subtasks). Scaffolds are often part of the task instruction.
feedback	Any form of feedback or evaluation by students, teachers or others on actions/statements/homework/etc. by students, teachers or other people. The feedback must be relevant to or part of the IBME activity.



Aspect	Definition
deciding on objectives	The process of deciding about objectives the group wants to work towards.
access to group	The readiness of the group to invite/welcome temporary visitors or new members.
discussions	The conversations that take place in group meetings and that may or may not follow a previously fixed agenda.
evaluation	The process of implementing some mechanism or approach to determine the group's success in achieving their common goals and the effectiveness of the group's collaboration.
reflection on professional growth	As a place of professional growth for the teachers participating, a group of inquiry can choose to explicitly think about or even promote the professional growth of their members.
reflection on teaching	The process of thinking about and evaluating past teaching experiences. It can be done individually in private or through exchanging and discussing experiences in a group.
joint planning of teaching	The preparation of course sessions, the design of courses and curricula, etc. – individually or in a group.
organisation of group work	The manner in which of the group of inquiry cooperates. The way in which the group's collaboration evolves.

Next, we describe in detail the quality nuances and illustrate some of them by examples.

Open & closed/& standardised/& structured/& formal/scientific. The quality nuance "open" generally refers to an absence of restrictions or a lack of guidance. The counterparts ("closed", etc.) tend to mean the opposite, namely that specific rules or circumstances limit the number of choices for an involved person and that advice or rules are given which specify what to do and/or how to do it. These quality nuances apply to the following aspects:

- *Spiderchart inquiry learning*: exploration (Table 3.1),[4] making observations, planning investigations, justifying (Table 3.2), asking questions, communication of findings (Table 3.3).
- *Spiderchart inquiry teaching*: feedback (Table 3.4), assessment format.
- *Spiderchart group of inquiry*: organisation of group work, access to group, discussions.

Quality nuance	Example-scenario corresponding to the selected quality nuance
open	Students may explore a question/topic/situation by consulting sources of their choice and using any strategies available.
quan./qual.	quan.—There are about as many situations of exploration of the type "open" as there are of the type "closed". qual.—Most situations of exploration are neither of type "open" nor of type "closed"; the freedom to explore is generally situated somewhere in the middle between the two.
closed	Students may explore a question/topic/situation in a very limited environment, for example testing certain types of input in a given program and observing what happens to the output, to form a hypothesis.

TABLE 3.1. Spiderchart *inquiry learning*, aspect *exploration*.

Quality nuance	Example-scenario corresponding to the selected quality nuance
open	How students justify their results or solve a problem is open for choice. Non-formal arguments are accepted as justifications.
quan./qual.	See Example 1 in Table 3.1.
closed	Students' justifications have to adhere to standards of formal mathematics or some scientific discipline.

TABLE 3.2. Spiderchart *inquiry learning*, aspect *justifying*.

[4]Regarding notation: Exemplary illustrations can be found in the given tables. Where no reference is made to a table the respective nuance does not have an illustration.

Quality nuance	Example-scenario corresponding to the selected quality nuance
open	Students can choose freely how to articulate or communicate their findings. (e.g., choosing a manner of presentation, creating a picture/diagram/poster, writing a proof/text/poem/etc.)
quan./qual.	See Example 1 in Table 3.1.
closed	Students have to follow some pre-structured format to articulate their findings. (e.g., multiple-choice questionnaire, writing a formal proof, using specific (technical) language or images, etc.)

TABLE 3.3. Spiderchart *inquiry learning*, aspect *communication of findings*.

Quality nuance	Example-scenario corresponding to the selected quality nuance
open	Feedback is given more or less spontaneously and in a rather unstructured manner. It is not oriented towards predefined criteria.
quan./qual.	See Example 1 in Table 3.1
closed	Feedback is given in response to specific contributions, e.g., as a fixed element of every student presentation or by students after every course/semester, and it is given in a structured manner, e.g., by going through a list of items to be commented on or by filling a questionnaire.

TABLE 3.4. Spiderchart *inquiry teaching*, aspect *feedback*.

Digital & analogue. If the interaction between teacher and students relies heavily on digital media, the quality nuance "digital" applies. The quality nuance "analogue" is used, if the interaction between teacher and students rely on analogue media. This quality nuance applies to the aspect *media* in the spiderchart *inquiry teaching* (Table 3.5).

Quality nuance	Example-scenario corresponding to the selected quality nuance
digital	STACK; internet; programming; etc.
quan./qual.	See Example 1 in Table 3.1.
analogue	Printed paper; blackboard; etc.

TABLE 3.5. Spiderchart *inquiry teaching*, aspect *media*.

Essential & non-essential. This pair of quality nuances refers to the necessity of actions in the context of the IBME activity. Is it essential for a student to do something in order to be able to adequately or successfully participate in the IBME activity? As a member of a group of inquiry, how essential are certain aspects of cooperation for the overall success of the group's project? The quality nuance "essential" is the only one for which the rule of thumb for the choice between "quan." and "qual." in the middle box does not apply. The rule that holds for this quality nuance is explained in Tables 3.6 and 3.7. These quality nuances apply to the following aspects:

- *Spiderchart inquiry learning*: cooperation, using tools.
- *Spiderchart group of inquiry*: deciding on objectives (Table 3.6), reflection on professional growth, reflection on teaching, joint planning of teaching (Table 3.7).

Quality nuance	Example-scenario corresponding to the selected quality nuance
essential	The group of inquiry discusses common objectives and changes objectives if necessary. Joint decision making with the involvement of all group members is considered essential for the group's work.
quan./qual.	quan.—The group discusses common objectives and/or changes objectives frequently. However, this is not considered essential for the group's work. qual.—Joint discussion of common objectives is considered to be essential for the group's work. However, it happens only very rarely.
non-essential	Common objectives are not discussed (anymore) in group meetings (e.g., objectives could be defined by one group member and followed by the others). Joint discussion of common objectives is not considered essential for the group's work.

TABLE 3.6. Spiderchart *group of inquiry*, aspect *deciding on objectives*.

Quality nuance	Example-scenario corresponding to the selected quality nuance
essential	The group of inquiry designs teaching units/materials together. The involvement of all group members in the planning of teaching is considered essential for the group's work.
quan./qual.	quan.—The group regularly plans teaching together. However, this is not considered essential for the group's work. qual.—Planning teaching as a group is considered essential for the group's work, but happens rather infrequently.
non-essential	Each group member has full responsibility for a course of their own. They may talk with other members about teaching, but each member designs their own course curricula, tasks, etc.

TABLE 3.7. Spiderchart *group of inquiry*, aspect *joint planning of teaching*.

Formative & summative. The quality nuance "formative" applies when the evaluation of an IBME activity focuses on the work process in the group of inquiry. The quality nuance "summative" applies when evaluation focuses on the outcomes of the group's work. This quality nuance applies to the aspect *evaluation* in the spiderchart *group of inquiry* (Table 3.8).

Quality nuance	Example-scenario corresponding to the selected quality nuance
formative	The group discusses regularly whether and how effectively/ easily goals have been achieved. If needed, the activity or the group's work organisation are changed.
quan./qual.	See Example 1 in Table 3.1.
summative	On a regular basis questionnaires or criteria checklists are used to determine if goals have been achieved.

TABLE 3.8. Spiderchart *group of inquiry*, aspect *evaluation*.

Weakly formatted & strongly formatted. If task instructions together with contextual information (e.g., didactic contract) do not or only slightly lead the solution process into a certain direction, the quality nuance of the aspect *task* is "weakly formatted". If task instructions in the given context hint at some intended solution or solution format, the quality nuance "strongly formatted" should be chosen. This quality nuance applies to the aspect *tasks* in the spiderchart *inquiry teaching* (Table 3.9).

Quality nuance	Example-scenario corresponding to the selected quality nuance
weakly formatted	A modelling problem that does not have fixed quality criteria for a solution and that allows for non-mathematical arguments and knowledge to be applied. This leaves room for a wide variety of different solutions or different types of solution.
quan./qual.	See Example 1 in Table 3.1.
strongly formatted	The task of formally proving a mathematical theorem limits the range of acceptable task solutions (arguments must adhere to specific quality standards, the language must adhere to specific norms of communication, there are correct and incorrect solutions, etc.).

TABLE 3.9. Spiderchart *inquiry teaching*, aspect *tasks*.

Reduced facilitation & increased facilitation. The quality nuance "reduced facilitation" applies, if no or very few scaffolds are used in a task instruction. The quality nuance "increased facilitation" applies when a broad array of scaffolds is provided in a task instruction. This quality nuance applies to the aspect *scaffolding* in the spiderchart *inquiry teaching* (Table 3.10).

Quality nuance	Example-scenario corresponding to the selected quality nuance
reduced facilitation	A typical Fermi problem does not guide the learner towards a specific solution path and does not provide much help.
quan./qual.	See Example 1 in Table 3.1.
increased facilitation	A mathematical task that consists of a sequence of consecutive subtasks, splitting the tasks in smaller, more manageable parts.

TABLE 3.10. Spiderchart *inquiry teaching*, aspect *scaffolding*.

Student-centred & teacher-centred. If a teaching format/approach relies on the active participation and engagement of students, it is "student-centred". If a teaching format/approach relies mostly on the teacher's actions and activities, it is "teacher-centred". This quality nuance applies to the aspect: *teaching methods* in the spiderchart *inquiry teaching* (Table 3.11).

Quality nuance	Example-scenario corresponding to the selected quality nuance
student-centered	A group project.
quan./qual.	See Example 1 in Table 3.1.
increased facilitation	A typical lecture.

TABLE 3.11. Spiderchart *inquiry teaching*, aspect *teaching methods*.

Student-chosen & teacher-chosen. If learners choose what problems to investigate and have control over the content studied, then the content (of the IBME activity or course) is "student-chosen". If a rigid curriculum exists that is followed very closely in the course of the IBME activity, the content is "teacher-chosen" (even if the curriculum was not designed by the teacher him-/herself). This quality nuance applies to the aspect *content* in the spiderchart *inquiry teaching* (Table 3.12).

Quality nuance	Example-scenario corresponding to the selected quality nuance
student-chosen	Group project with relatively open choice of question.
quan./qual.	See Example 1 in Table 3.1.
teacher-chosen	The contents of a specific book have to be studied in a fixed amount of time.

TABLE 3.12. Spiderchart *inquiry teaching*, aspect *content*.

Applied & theoretical. The quality nuance "applied" is used, if the type of mathematics students and teachers inquire into is essentially shaped by its context of use. The quality nuance "theoretical" is used, if the inquiry focuses on innermathematical topics. This quality nuance applies to the aspect *type of mathematics* in the spiderchart *inquiry teaching* (Table 3.13).

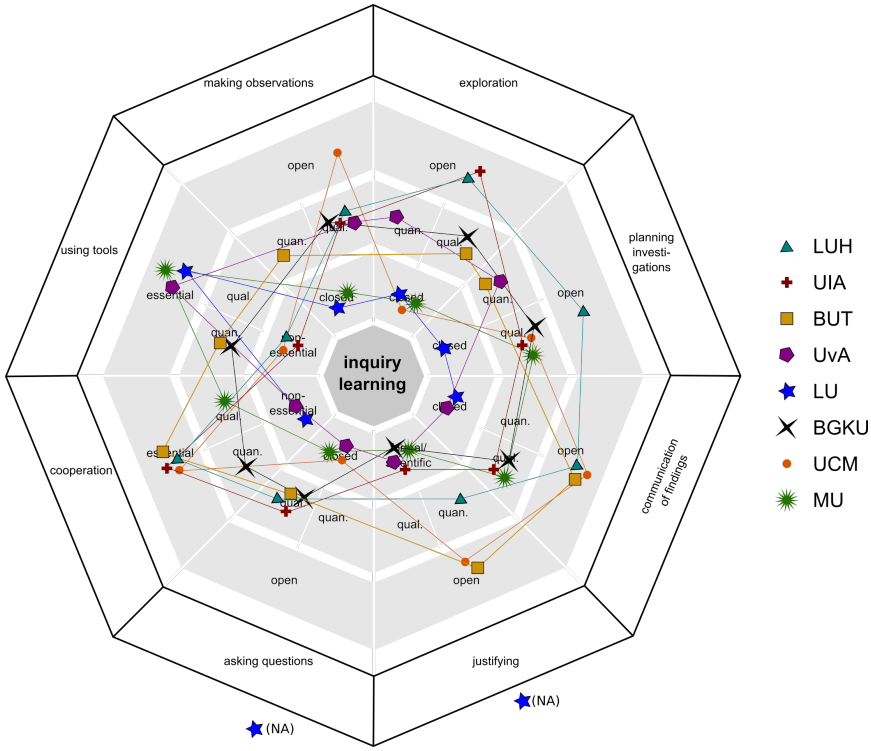
Quality nuance	Example-scenario corresponding to the selected quality nuance
applied	Mathematical modelling; mathematics teaching scenarios.
quan./qual.	See example 1 in Table 3.1
theoretical	Any topic of pure mathematics; foundations of mathematics.

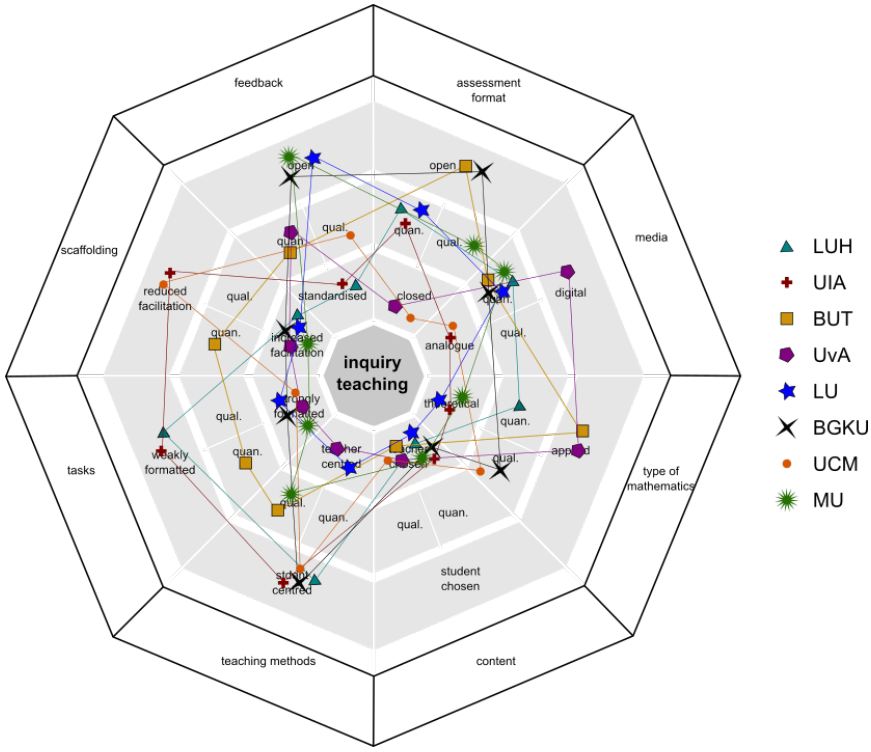
TABLE 3.13. Spiderchart *inquiry teaching*, aspect *type of mathematics*.

3.4. The Spidercharts of the PLATINUM Cases

In order to fill in the spidercharts with each of our PLATINUM partners, we organised online meetings of about an hour each. These meetings served to assist the partners in filling in the spidercharts and to clarify ad hoc questions. Before the meeting, we sent an email to our partners asking them to look at the charts and to look through a preliminary manual providing more detailed information about the aspects and nuances than we have just presented in this text. In addition, we asked the partners to recall their local case study or an example from it that was suitable for filling in the charts. Thus, the completed spidercharts represent a structured survey of the partners' understanding and interpretation of the different aspects and quality nuances in view of their respective case studies.[5] The spidercharts do not serve us here to evaluate the individual activities or to classify whether inquiry has taken place or not. Instead they illustrate the wide variety of possible inquiry activities. We integrated the choices of all individual partners into one chart (see Figure 3.1, spread over the next two pages). In this way we can graphically illustrate similarities and differences between the different local cases of our partners of the PLATINUM project. In the following we describe some initial observations. More detailed analyses are of course possible but will be reported elsewhere.

[5]The spidercharts are primarily designed as a reflection tool for the groups and in the interaction between groups. The activity of filling in, discussing and reflecting on the individual aspects within the group is essential. The completed spidercharts themselves are rather insignificant with regard to this function. The completed spidercharts also do not represent "objective" scientific data and should not be used to assess or evaluate the groups or the associated case studies or IBME activities. A comparison is also only possible to a limited degree, as the aspects and quality-nuances are open and require the interpretation of the respective partners in their context.


making observations
exploration
using tools
planning investi-gations
cooperation
communication of findings
asking questions
justifying
inquiry learning
open
quan.
qual.
closed
essential
non-essential
(NA)
LUH
UIA
BUT
UvA
LU
BGKU
UCM
MU


feedback
assessment format
scaffolding
media
tasks
type of mathematics
teaching methods
content
inquiry teaching
open
quan.
qual.
standardised
closed
digital
analogue
reduced facilitation
weakly formatted
teacher centred
student chosen
(NA)
LUH
UIA
BUT
UvA
LU
BGKU
UCM
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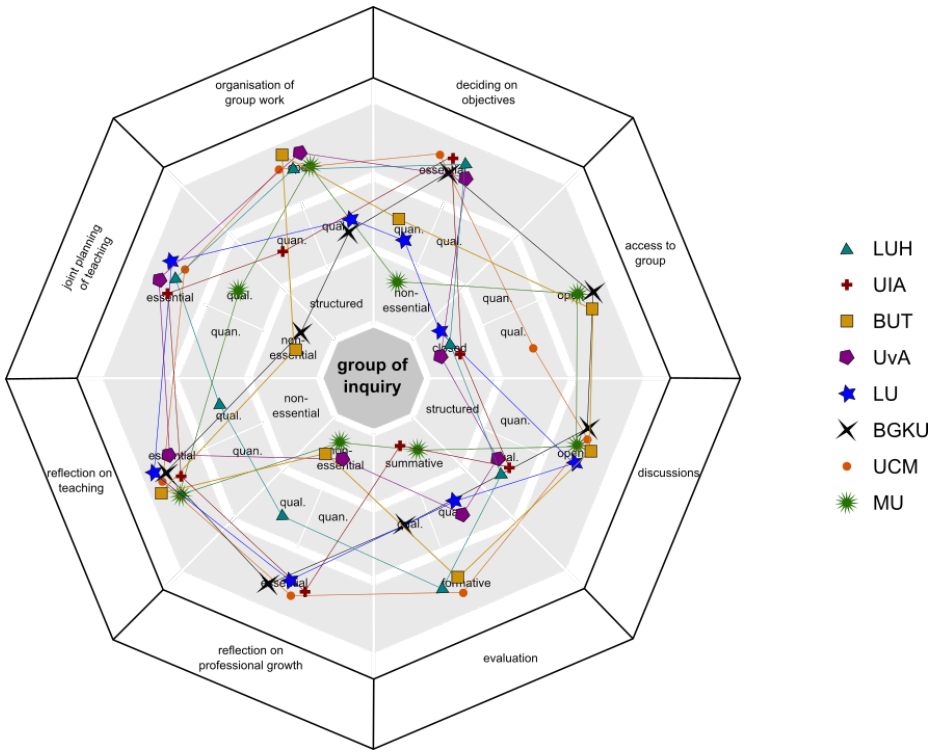

FIGURE 3.1. Spidercharts with options chosen by the PLATINUM partners (coloured version in the ebook).

If we take a look at the spiderchart *inquiry learning*, it is striking that no partner chose *asking questions*[6] as "open." For most partners students' cooperation was considered to be essential for the IBME activities. Only the partners from UvA and LU considered "cooperation" as "non-essential." Instead both partners report about the importance of their computer-based setting. It also seems to be due to their general institutional conditions that usually lecture rooms are equipped with individual computer workstations, which makes cooperative approaches at least more difficult. The issue of individual computer workstations for IBME activities was also discussed by BUT. Designs of lecture rooms that are equipped for computer-based group work are rare (Rønning, 2019). The aspects *using tools*, *communication of findings*, and *exploration* show a great variance, which reveals a scope for didactic decisions.

Considering *inquiry teaching* it is noticeable that, in comparison to the other spidercharts, for many aspects quality nuances in the inner ring are chosen. The aspects *content*, *teaching methods*, *tasks*, and *scaffolding* form a cluster that could be described as a type of "guided teaching": teacher chosen content, teacher centred teaching methods, strongly formatted tasks that guide the inquiry process together with increased facilitated scaffolding. The combination of those quality nuances guide the IBME activities in a desirable and manageable direction. For further explanation, we will discuss the options chosen by the UiA as an example of this type and contrast the

[6]The partners from Loughborough considered the aspects *asking questions* and *justifying* to be inapplicable to their case. In their setting students did not communicate openly their questions and justifications during the IBME activity.

options chosen by the UvA with them: UiA describes two cases for inquiry teaching. They chose a rather unconventional teaching scenario by deliberately choosing "unusual situations." One attempt is valued as a successful implementation and one attempt was abandoned. LUH has chosen a similar task design and also reports difficulties and resistance in implementation due to the given institutional conditions. UvA, in their case study, also points to several but different challenges that led them to opt for a more "guided" teaching-approach: Firstly, they refer to challenges that are rooted in the digital environment. The digital environment seems to be very challenging for their students, which made more scaffolding and a teacher-centred approach necessary.[7] Secondly, they draw on different types of mathematical content (statistics and mathematics) and its instrumental genesis.[8]

In contrast to the spiderchart *inquiry teaching*, the spiderchart *group of inquiry* has many options chosen in the outer ring. Two types of groups emerge: Firstly, "planning groups" that are not open in access and tend to reflect on professional growth within the group. And "open groups" who do not see planning as an essential group activity and who generally do not consider reflecting on professional growth as an essential group activity. The options chosen by BGKU show that it is also possible to choose a design for inquiry group activities that is open in access and provides spaces for reflecting professional growth collectively. MU does not reflect professional growth collectively. The group activities described in the case study show a strong connection to individual reflection of professional growth. UCM can be considered to be a planning group. They describe several inquiry teaching scenarios for different types of courses in their case study. In their work, reflection on professional growth takes place through joint theory-based planning.

3.5. Discussion

The spidercharts intend to support reflection processes revolving around the concepts of inquiry learning and inquiry teaching and the work in groups of inquiry. The quality nuances of their aspects represent different locations along a continuum. However, they are not prescriptive of what constitutes an appropriate choice: Per se, none of these options are better or worse choices to foster IBME or to bring forward the work in our local groups of inquiry. Instead of categorising, the charts are intended to inspire reflection processes: The forced choice of one out of four options along the continuum motivates reflection on aims of, choices made in, broader conditions of (etc.) local IBME activities. The charts do not intend to serve as an evaluation tool that judges the "success" of a local group or IBME activity. In the best case, they help to reveal hitherto unconsidered opportunities for intervention and to make hitherto implicit decisions explicit with regard to the respective local cases.

The interviews we conducted on the occasion of filling in the spidercharts showed us that the three charts actually cover a very wide range of possible IBME activities. There was no case that could not be located in terms of important aspects from the point of view of the interview partners. Moreover, the results really show the great diversity of IBME activities developed and implemented in the PLATINUM project. Thus, central goals of our development of the spidercharts could be achieved.

[7]From the UvA case study: "This teacher direction is called instrumental orchestration in the instrumental approach" (p. 227).

[8]From the UvA case study: "We also had not realised that the instrumental genesis of students during the statistic part of the course had a different orientation than the one needed for the tool use in the module" (p. 227).

In summary, the spidercharts could help our project partners and us to articulate ideas, to adequately characterise each of the various contributions in this project, to grasp and compare creative leeways and thus to express specificities of the local cases within PLATINUM. Thus, we believe that the spidercharts are helpful to structure the presentations of our work in the PLATINUM project and to facilitate communication between local groups of inquiry. Moreover, a first rough comparison already yields some results that appear typical, for example with regard to the inner or outer concentration of ticks on the various spidercharts. It has to be understood, though, that behind similar lines there could be different situations or reasons for didactic-methodical decisions. This suggests that very narrow interpretations of the content of trajectories should be taken with caution. For the time beeing, we will leave open the question of whether further conclusions can be drawn from more detailed analyses of the completed spidercharts. Overall, we believe that the spidercharts are helpful to structure the presentations of our work in the PLATINUM project and to facilitate communication between local groups of inquiry.

At the same time, the spidercharts can easily be applied as reflection tool outside of PLATINUM, by any (group of) teacher(s) that wishes to engage in developing their teaching further in the direction of enhanced inquiry-orientation. Also, the charts are not strictly research-oriented, in the sense that they are intended to be integrated in or followed by a developmental research project. Their usage can pursue aims not related to research in a narrow sense.

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