Inquiry in University Mathematics Teaching and Learning: The PLATINUM Project
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CHAPTER 1

Introduction

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Mathematical demands in university courses represent a considerable hurdle for many students and for quite a few this is known to lead to dropping out or changing studies. But even among students who successfully pass mathematics exams, there are many who do not achieve the intended teaching goals. Instead of a reflected understanding of mathematical concepts and their internal and external mathematical use, rote learning often dominates. Concepts such as complex numbers or derivatives of functions are technically mastered, which is certainly important, but meanings that go beyond this, for example, reasoning of properties or relationships, are not acquired. If one does not want to attribute these results one-sidedly to deficits in the students, such as a lack of talent or commitment, then one must question the dominant teaching and also the prevailing examination practices.

IBME is a central approach to designing teaching differently. It aims directly to ensure that students are not only presented and shown some mathematics and subsequently trained in techniques, but are involved much more in the teaching-learning context from the very beginning. In the best case, for example, mathematical concepts should be (re-)discovered starting from a problem as an answer to a question. This intention is to regard teaching and learning of mathematics as closer to how mathematicians proceed in research itself and less like how mathematics lessons are regarded traditionally. This is why it is not uncommon to speak of research-oriented teaching.

The European project PLATINUM (Partnerships for Learning And Teaching in University Mathematics), a consortium of 8 universities from 7 countries (University of Agder, Norway; University of Amsterdam, Netherlands; Masaryk University, Czech Republic; Brno University of Technology, Czech Republic; Loughborough University, England; Leibniz University Hannover, Germany; Complutense University of Madrid, Spain; Borys Grinchenko Kyiv University, Ukraine) has set out to develop Inquiry-based mathematics education (IBME) approaches in the teaching of mathematics. For this purpose, the partners made their way to create a community that exchanges ideas about possibilities and limits of the implementation and realization of the different design approaches of IMBE. The partners reflect these approaches critically against respective local practical and theoretical backgrounds and, in particular, create a space for the creative development of IBME-oriented teaching units, tasks and further training. The interlocking of three levels: the learning activities of the students, the planning and design of the teaching by the teachers, and their critical academic reflection and monitoring represent, in a sense, a brand core of PLATINUM (see the Three-Level Model in Chapter 2). In this way, the project partners themselves entered into an inquiry process and formed a community of inquiry as described by Jaworski (2019).

Of course, some general problem areas of didactics played a role here as well, which we will briefly discuss in the following. A first consists of close collaboration between didacticians and mathematicians. As fruitful and necessary as such cooperation is in practice, especially for PLATINUM, it also involves different perspectives, which can be seen as complementary but also as potentially conflictual. Clearly, didacticians and mathematicians pursue the same goal, that students understand mathematics better and more deeply, but to a certain extent they speak different languages and embody different scientific cultures. In particular, their scientific discourses follow different norms of problem formulation and justification of answers to questions. Mathematicians, for example, tend to demand unequivocal evidence or proof for hypotheses and are on the search for methods to obtain such evidence through systematic procedures. Didacticians are usually aware that this must fail, that already in questions and problem formulations theories and ideas are implicitly included, which are to a certain extent unprovable, raise questions themselves, but this does not mean on the other hand, of course, that didactic research would be completely arbitrary and that justifications would not be based on rationales.

Clearly, mathematicians want clear evidence that IBME leads to better student learning outcomes. It is understandable that it is not satisfying when didacticians point out that this question addresses a major problem in a somehow undercomplex and problematic way. To outline just one aspect of this problem: Luhman and Schorr (1982) pointed out the so-called technology deficit of pedagogy. By this they meant the fundamental and insurmountable difficulty of a lack of linear causality between, say, a teacher's intention and the effect that actually occurs with learners. Learning processes can only ever be stimulated, but never directly achieved. Of course, this does not speak against efforts to make teaching more successful and to search for conditions that make desired learning possible or to question conditions that prevent it. However, concrete instructions for action with necessarily occurring learning effects are not possible and scientifically justifiable. Of course, mathematicians in general are aware of this, but usually not with regard to all the problems that arise from it in terms of scientific concepts of didactics and, for example, methods of research and evaluation.

This is one reason why didacticians, like the authors of this introduction, tend to speak of teaching goals rather than learning goals, see above. Regarding teaching goals or learning outcomes, the former are central to the teaching that takes place and therefore influence the learning that takes place (i.e., the learning outcomes). Beyond what has been said so far, unfortunately 'learning outcomes' are often expressed in a general way by those writing the curriculum and often do not match the pedagogies and the goals that teachers have for their students through the teaching-learning interactions which take place.

In particular, Holzkamp (1995) has shown that, to a certain extent, under societal pressure on educational institutions, learning theories and curricular tend to teaching-learning shortcuts, i.e., to speak in fact about teaching but attempt to formulate this in terms of learning outcomes. These then also somehow suggest that empirical research should be established with the aim of finding clear justifications for concrete instructions for teaching actions that directly ensure learning outcomes. As important and significant as this research is, a different understanding of human learning, such as in Lave and Wenger (1991), also resulted from and justifies the partial failure of this research logic. Their concept of Community of Practice also underlies PLATINUM. However, a deeper understanding of this concept, as simple as it may seem at first, is not so easy to gain. In particular, it requires an understanding of the problematic situations just outlined, not as clumsiness that can be easily overcome, but as unresolvable fields of tension in which efforts for better teaching and also IBME necessarily have to proceed.

Summarising, actions linked to teaching goals aim at learning goals, but these just cannot be ensured. Nevertheless, we believe that there are good reasons for teaching in terms of IBME. Diverse reasons and corresponding suggestions for the design of such teaching can be found in many places in this volume. In particular, this involves subject-specific proposals that are concretised in tasks and learning situations, for example. And this is precisely what makes high subject-specific demands in the field of university mathematics and cannot be mastered without mathematics experts, since general didactic considerations are not sufficient here. This is exactly the point where the complementarity of didactics and mathematics has to prove itself. A teaching design oriented towards IBME requires a different preparation of the material to be taught. It is oriented more towards questions and problems that mathematics deals with and answers, and less towards the answers themselves. The questions and problems, and how they can be approached through suitable tasks and materials, require a really good subject-specific and, above all, flexible understanding of the mathematical content. In many chapters it becomes clear that mathematics experts were active in PLATINUM. And this is especially true for the many proposals related to mathematical service-courses. Here, the authors must not only have a high command of mathematics, but also an understanding of the service subject, such as engineering, economics, or biology. Mathematics is used there in a specific way. Symbols etc. take on additional meanings and practices are, partly, justified differently, precisely from the respective subject-specific context. In other words, mathematics is not simply applied, but changes in a specific way in its use in other sciences, especially empirical sciences.

A special role is played by proofs. Clearly, the role of proof in mathematics is different from the role of proof in the didactics of mathematics, as explained above. As we inquire into mathematical processes and make our own conjectures, we aspire to mathematical proof, just as mathematicians do. However, the theory of inquiry operates ‘around’ the mathematics that we do. In terms of the three-layer model, mathematical proof would be a part of the central layer with students and their teachers. How to achieve such mathematical proof in teaching and learning, through inquiry, is part of the second layer as teachers make sense of inquiry approaches to explore mathematics and achieve mathematical proof. The design of teaching units takes this into account. The outer layer formalises the processes of the second layer, striving for a cohesive account of the developmental processes in the second layer and the overall complex process in the three layers.

Against this background, the present volume is a major outcome of our inquiry processes. It presents both basic theoretical concepts that accompanied our process and its organisation as well as the writing of this volume (Part 1, Chapter 2). In addition, Part 1 includes the presentation and reflection of an IBME-orientated tool developed in the project, the so-called ‘spidercharts’, as well as that of a general model for the consideration of students with indentified needs in IMBE-oriented teaching. Part 2 of the volume then focuses more concretely on the project as a whole: starting with the organisation and implementation of the project (Chapter 5). This is followed by concepts and examples of the development of tasks and teaching units (Chapter 6) and of the implementation of corresponding professionalisation approaches in inquiry-based teaching and learning for lecturers (Chapter 7). In Chapter 8, the development of tasks and teaching units is taken further with a focus on mathematical modelling.

The evaluation of IBME teaching is not straightforward and cannot be done through traditional quantitative instruments or essentially procedural-orientated examinations, but requires specific instruments orientated at the goals of IBME. Although there is certainly still a lot of development work and research to be done, Chapter 9 presents important ideas that can be implemented in practice.

The third and major part of the volume consists of so-called 'case studies.' After an introduction (Chapter 10), each partner describes and reflects on concrete examples of their own development within the project. According to the respective professional backgrounds and local conditions and possibilities, the contributions range from more theoretically oriented reflections to interesting practical presentations. In our view, the diversity presented here has been one of the great strengths of the project. It becomes especially clear that inquiry does not live from standardisation but from heterogeneity and different perspectives, from freedom instead of superficial normalisation.

In the final 4th part we summarise once more: Where did we start and where did we end up? What are the key experiences and insights from our project? What perspectives are emerging? We consider the journey we have taken in interpreting inquiry-based learning and teaching in PLATINUM and synthesising our activity and learning for this book.

For whom have we written this book? For ourselves, first of all. Describing our local developments, incorporating our experiences into the more theoretical parts, and reflecting on the experiencing and design of IBME approaches against a theoretical background, this interweaving of practice and theory could be taken to a new level through writing, reviewing and optimising. And then, of course, we think that what we have written here can also be interesting and fruitful for people outside. In particular the case studies offer interesting insights and suggestions for practitioners who are interested in didactic questions as well as for researchers working in the field of didactics. People interested in IBME could find answers or at least ideas regarding questions about the evaluation of measures, systematic ways of designing learning units and tasks and in doing so also taking into account special needs. Last but not least, our own professionalisation through inquiry can be seen as a basis for professionalisation more widely involving approaches to mathematics teaching and learning that benefit our students.

If we could wish for something in conclusion, it would be that this volume contributes to stimulating the development and preparation of well-reflected and evaluated IBME teaching units and tasks and to optimising existing ones. Last but not least, we also think of the teachers, mathematics colleagues, for whom teaching with more successful students also brings pleasure and contentment.

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